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A Comparative Evaluation of Pitch Notations in Turkish *Makam* Music:

Abjad Scale & 24-Tone Pythagorean Tuning – 53 Equal Division of the Octave as a Common Grid

Türk Makam Müziği'nde Perde Notasyonaların Karşılaştırmalı Bir Değerlendirmesi: Ebced Dizisi ve Gayri Müsavi 24 Perdeli Taksimat – Oktavın 53 Eşite Dilimlenmesiyle Elde Edilen Ortak Bir Izgara

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Abstract. In the Middle East, *Abjad* notation has been available since the day of *Al-Kindi* (9th century). However, *Abjad* was never used except as a theoretical tool. A handful of extant musical examples show that *Abjad* did not appeal to the general body of composers and executants throughout the ages, but was confined to treatises as a means of explaining and demonstrating the ability to notate pitches. This article features a comparative evaluation, apparently for the first time, of two historical notations in Turkish *makam* music based on *Abjad*. These are, Safi al-din Urmavi's 17-tone Pythagorean tuning (13th century) and Abd al-Baki Nasir Dede's attribution of *perde* (tone/fret) names to the same (19th century). The juxtaposition of *Abjad* Scale side by side with the current theory of Turkish *makam* music known today as *Arel-Ezgi-Uzdilek* (AEU) proves that the latter is simply an extension of Urmavi's archetype. This emphasis constitutes one of our contributions. Overall, 53 equal divisions of the octave is found to embrace them with less than a cent error, although this resolution has never been fully utilized in Turkish *makam* music.

Keywords: *Abjad* Scale, Pythagorean Tuning, *Arel-Ezgi-Uzdilek* System, 53 equal divisions of the octave

Özet. *Ebced* notalama sistemi, *Al-Kindi*'nin yaşadığı dönemden (9.yy) beri Ortadoğu'da bilinmektedir. Ancak, *Ebced*, sadece kuramsal bir araç olarak kullanılmıştır. Günümüze ulaşan sınırlı sayıda müziksel örnekten görmek mümkündür ki, *Ebced*, çağlar boyunca bestekarların ve icracıların geneline hitap etmemiş, edvar/nazariyat kitaplarında perdeleri açıklama ve simgeleme yolu olarak kalmıştır.
Bu makale, görüldüğü kadarıyla ilk kez olarak, *Ebced*'e dayalı tarihi iki Türk *makam* müziği notasının karşılaştırmalı değerlendirmesini içermektedir. Bunlar, Safiyüddin Urmevi'nin 17-sesli Pithagorsal düzeni (13. yy) ve Abdülbaki Nasır Dede'nin ona *perde* isimleri vermesiyle oluşan kurgudur (19. yy). Bugün *Arel-Ezgi-Uzdilek* olarak bilinen yürürlükteki Türk *makam* müziği kuramı ile *Ebced* dizisi yan yana konduğunda, ilkinin Urmevi tarafından geliştirilen ana-modelin devamı olduğu görülür. Bu olgunun vurgulanması bizim bir katkımızdır. Son toplamda, Oktavın 53 eşit parçaya bölünmesi, bunları bir sentin altında hata ile sarmalamaktadır. Ancak, bu çözünürlük Türk *makam* müziğinde bütünüyle uygulanmamaktadır.

Anahtar sözcükler: *Ebced* dizisi, Pithagorsal Düzen, *Arel-Ezgi-Uzdilek* Sistemi, 53-ton Eşit Taksimat

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1 Introduction

This study investigates the commonalities between the historical Abjad scale and the 24-tone Pythagorean Model currently in use in Türkiye.

Abjad, which is the Arabic shorthand for "ABCD", was initially a guide to learning the Arabic alphabet and pronounciation of letters by rote, yet, gradually developed into numerology and a method of calculating dates (Ekmekçioğlu 1992, 16-33; Tura 1982 (1998), 178) as seen below in Table 1.

Table 1. Arabic Letters vs Numbers

ا	ب	ج	د	ه	و	ز	ح	ط	ي
1	2	3	4	5	6	7	8	9	10
ك	ل	م	ن	س	ع	ف	ص	ق	
20	30	40	50	60	70	80	90	100	
ر	ش	ت	ث	خ	ذ	ض	ظ	غ	
200	300	400	500	600	700	800	900	1000	

Abu Yusuf Yaqub ibn Ishaq Al-Kindi, premier Abbasid philosopher, who lived ca.800-873 C.E. (El-Ehwany 1961), was the first to utilize *Abjad* as a pitch notation (Turabi 1996). Centuries later, Abbasid scholar *Safi al-din Abd al-mu'min Urmavi* (1216-1294) revived *Al-Kindi*'s *Abjad* and revised it to notate his unique 17-tone Pythagorean scale (Uygun 1999; Çelik 2004). *Abd al-Qadir Meragi* (ca.1360-1435) also employed Urmavi's scale in his tractates (Bardakçı 1986). *Nur al-din Abd ar-Rahman Djami* (1414-1492) copied his predecessor (Djami ca.1450 (1965)), after whom, a quadricentennial epoch deserving to be titled "the Dark Ages of *makam* theory" prevailed – during which time mathematical calculation of pitches lapsed.

By the end of the 18th century, *Abd al-Baki Nasir Dede* (1765-1821) introduced a modified *Abjad* notation (IRCICA 2003: 130-4) just decades before the awakening in musical arithmetics took place.

By 1910, Rauf Yekta conceived on staff a 24-tone Pythagorean tuning that was none other than the continuation of *Urmavi*'s scale (Yekta 1922: 57-9). Later on, Yekta's contribution was revamped by his peers Saadettin Arel, Suphi Ezgi, and Murat Uzdilek, and has been taught since in Turkish Music conservatories under the name of "*Arel-Ezgi-Uzdilek*" (Öztuna 1969: 45-61, 205-9).

Because of the excellent proximity of either 24-tone model to the related tones of 53-equal divisions of the octave, the "9 commas per whole tone; 53 commas per octave" methodology is unanimously accepted in Turkish *makam* music parlance and education.

In this article, we are going to compare *Safi al-din Urmavi*'s 17-tone scale (13th century) and *Abd al-Baki Nasir Dede*'s Abjad notation (19th century) with the 24-tone

Pythagorean Model in force today. Our conclusion will be that the latter is simply an extension of the former, all of which can be represented with less than one cent error in 53-tone equal temperament, although, observedly, this resolution is not implemented as a whole on any actual instrument of Makam Music.

2 *Al-Kindi*'s Ud Fretting and *Abjad* Notation

Muslim philosopher *Al-Kindi* was the first to make use of *Abjad* to denote finger positions on the ud. Though he mentioned Greek tetrachordal genera involving the division of the whole-tone into quarters, his 12-note approach is purely Pythagorean (Turabi 1996: 88-92), and is the precursor to *Urmavi*'s scale, as shown in Figure 2 and Table 2.

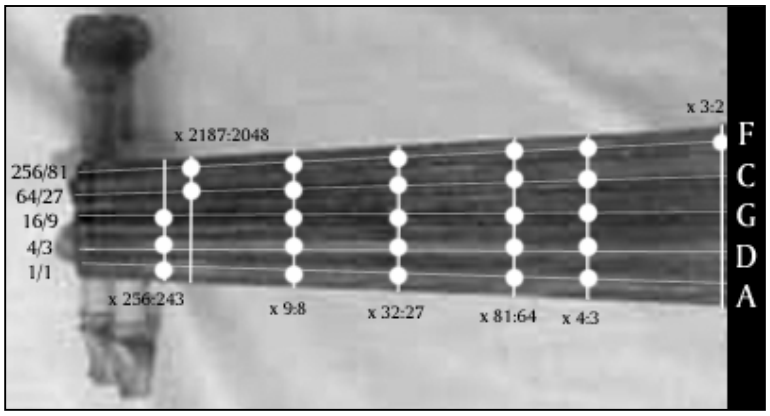


Figure 2. *Al-Kindi*'s Ud Fretting

Table 2. *Al-Kindi*'s Ud Fingering and *Abjad* Notation [1]

Open String	A **1/1** ا	D **4/3** و	G **16/9** ك	C **32/27** د	F **128/81** ط
x 256:243	Bb **256/243** ب	Eb **1024/729** ز	Ab **4096/2187** ل	(Db) 8192/6561	(Gb) 32768/19683
x 2187:2048	(A#) 2187/2048	(D#) 729/512	(G#) 243/128	C# **81/64** هـ	F# **27/16** ي
x 9:8	B **9/8** ج	E **3/2** ح	A **2/1** ا	D **4/3** و	G **16/9** ك
x 32:27	C **32/27** د	F **128/81** ط	Bb **256/243** [2] ب	Eb **1024/729** ز	Ab **4096/2187** ل
x 81:64	C# **81/64** هـ	F# **27/16** ي	B **9/8** ج	E **3/2** ح	A **4/1** ا
x 4:3	D **4/3** و	G **16/9** ك	C **32/27** د	F **128/81** ط	Bb **256/243** ب
x 3:2	(E) 3/2	(A) 2/1	(D) 4/3	(G) 16/9	B **9/8** ج

3 *Urmavi*'s 17-Tone Scale

Late Abbasid scholar *Safi al-din Abd al-mu'min Urmavi* proposed for the first time in history a unique 17-tone scale reminiscent of *Al-Kindi*'s, which he notated using *Abjad* (Uygun 1999; Çelik 2004). He constructed it via a concatenation of 4 pure fifths up and 12 fifths down from an assumed tone of origin (5 additional fifths down compared to *Al-Kindi*), as shown in Table 3 and Figure 3.

[1] For the sake of simplification, I have chosen not to burden the reader with needless Arabic appellatives for ud strings and frets.

Table 3. Chain of Fifths Making *Urmavi*'s 17-tone Scale

Fifths	**Frequency Ratios**	**Octave Normalization**		**Classic Interval Names**
$3^4 : 2^4$	81/16	7.	81/64	Pythagorean major third
$3^3 : 2^3$	27/8	14.	27/16	Pythagorean major sixth
$3^2 : 2^2$	9/4	4.	9/8	major whole tone
3 : 2	3/2	11.	3/2	perfect fifth
0	1/1	1.	1/1	(tone of origin – perfect prime)
2 : 3	2/3	8.	4/3	perfect fourth
$2^2 : 3^2$	4/9	15.	16/9	Pythagorean minor seventh
$2^3 : 3^3$	8/27	5.	32/27	Pythagorean minor third
$2^4 : 3^4$	16/81	12.	128/81	Pythagorean minor sixth
$2^5 : 3^5$	32/243	2.	256/243	limma, Pythagorean minor second
$2^6 : 3^6$	64/729	9.	1024/729	Pythagorean diminished fifth
$2^7 : 3^7$	128/2187	16.	4096/2187	Pythagorean diminished octave
$2^8 : 3^8$	256/6561	6.	8192/6561	Pythagorean diminished fourth
$2^9 : 3^9$	512/19683	13.	32768/19683	Pythagorean diminished seventh
$2^{10} : 3^{10}$	1024/59049	3.	65536/59049	Pythagorean diminished third
$2^{11} : 3^{11}$	2048/177147	10.	262144/177147	Pythagorean diminished sixth
$2^{12} : 3^{12}$	4096/531441	17.	1048576/531441	Pythagorean diminished ninth

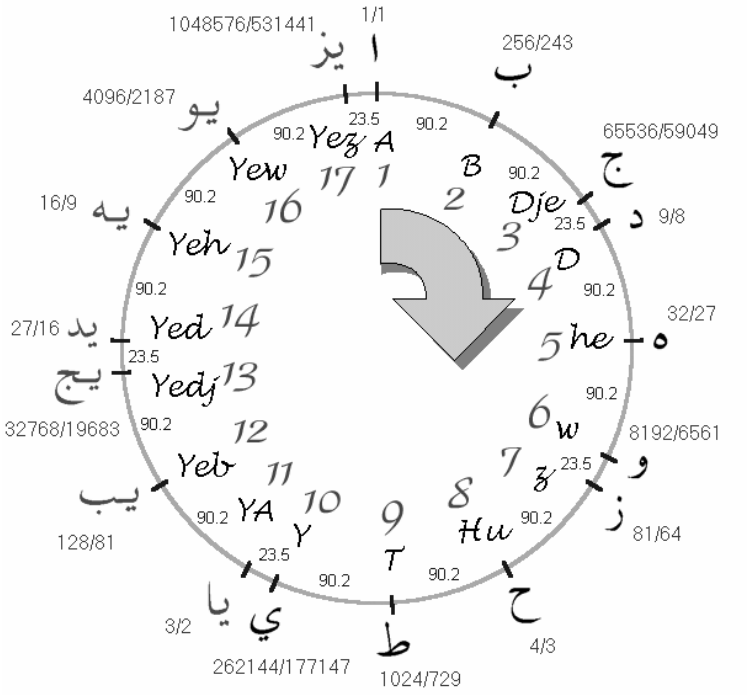


Figure 3. *Urmavi*'s 17-tone Pythagorean System

4 Comparison of *Urmavi*'s *Abjad* Scale with *Nasır Dede*'s Usage

The *Abjad* Scale of *Urmavi* following the pattern ابجد هوز حطي originally spanned two octaves. *Abd al-Baki Nasir Dede* extended the gamut by a whole tone and labelled its *perdes* (Başer (Aksu) 1996: 39-42) as seen in Table 4.

Table 4. Complete *Abjad* Notation of *Perdes* [2]

Abjad	*Urmavi* Ratios	Cents	Intervals	Degrees		*Nasir Dede Perdes*
A	**1/1**	**0.000**		ا	①	**Yegah**
B	256/243	90.225	90.225 ¢	ب	②	Pes Beyati
Ce	65536/59049	180.450	90.225 ¢	ج	③	Pes Hisar
D	**9/8**	**203.910**	23.460 ¢	د	④	**Aşiran**
he	32/27	294.135	90.225 ¢	ه	⑤	Acem Aşiran
Ve	**8192/6561**	**384.360**	90.225 ¢	و	⑥	**Arak**
Z	81/64	407.820	23.460 ¢	ز	⑦	Gevaşt
Hu	**4/3**	**498.045**	90.225 ¢	ح	⑧	**Rast**
T	1024/729	588.270	90.225 ¢	ط	⑨	Şuri
Y	262144/177147	678.495	90.225 ¢	ي	⑩	Zirgule
YA	**3/2**	**701.955**	23.460 ¢	يا	⑪	**Dügah**
YeB	128/81	792.180	90.225 ¢	يب	⑫	Kürdi/Nihavend
YeC	**32768/19683**	**882.405**	90.225 ¢	يج	⑬	**Segah**
YeD	27/16	905.865	23.460 ¢	يد	⑭	Buselik
Yeh	**16/9**	**996.090**	90.225 ¢	يه	⑮	**Çargah**
YeV	4096/2187	1086.315	90.225 ¢	يو	⑯	Saba
YeZ	1048576/531441	1176.540	90.225 ¢	يز	⑰	Hicaz/Uzzal
YaH	**2/1**	**1200.000**	23.460 ¢	يح	⑱	**Neva**

[2] *Perdes* expressed in bold are diatonic naturals.

Table 4. Complete Abjad Notation of *Perdes* – Continued

Abjad	*Urmavi* Ratios	Cents	Intervals	Degrees		*Nasir Dede Perdes*
YaH	**2/1**	**1200.000**	23.460 ¢	يح	⑱	**Neva** (ع)
YaT	512/243	1290.225	90.225 ¢	يط	⑲	Beyati
ke	131072/59049	1380.450	90.225 ¢	ك	⑳	Hisar
kÂ	**9/4**	**1403.910**	23.460 ¢	كا	㉑	**Hüseyni**
keB	64/27	1494.135	90.225 ¢	كب	㉒	Acem
keC	**16384/6561**	**1584.360**	90.225 ¢	كج	㉓	**Evc**
keD	81/32	1607.820	23.460 ¢	كد	㉔	Mahur
keh	**8/3**	**1698.045**	90.225 ¢	كه	㉕	**Gerdaniye**
keV	2048/729	1788.270	90.225 ¢	كو	㉖	Şehnaz
keZ	524288/177147	1878.495	90.225 ¢	كز	㉗	††
kaH	**3/1**	**1901.955**	23.460 ¢	كح	㉘	**Muhayyer** (كز) ㉗
keT	256/81	1992.180	90.225 ¢	كط	㉙	Sünbüle (كح) ㉘
L	**65536/19683**	**2082.405**	90.225 ¢	ل	㉚	**Tiz Segah** (كط) ㉙
LÂ	27/8	2105.865	23.460 ¢	لا	㉛	Tiz Buselik (ل) ㉚
LeB	**32/9**	**2196.090**	90.225 ¢	لب	㉜	**Tiz Çargah** (لا) ㉛
LeC	8192/2187	2286.315	90.225 ¢	لج	㉝	Tiz Saba (لب) ㉜
LeD	2097152/531441	2376.540	90.225 ¢	لد	㉞	Tiz Hicaz (لج) ㉝
Leh	**4/1**	**2400.000**	23.460 ¢	له	㉟	**Tiz Neva** (لد) ㉞
LeV	1024/243	2490.225	90.225 ¢			Tiz Beyati (له) ㉟
LeZ	262144/59049	2580.450	90.225 ¢			Tiz Hisar (لو) ㊱
	9/2	**2603.910**	23.460 ¢			**Tiz Hüseyni** (لز) ㊲

†† The octave complement of *zirgule* does not exist in *Nasir Dede*, and is therefore skipped.

In *Nasir Dede*, "*Pes*" (bass) signifies pitches an octave low, and "*tiz*" (treble) signifies pitches an octave high. The octave complement of *perde zirgule* does not exist. The letter *'ayn* is employed for degrees 18 and 28 instead of *Urmavi*'s *y*. Although, *Nasir Dede* also notated *makam* music *perdes* in *Abjad*, the sheik did not specify any ratios. His approach is compatible with the flexible nature of his instrument, the ney, which can produce subtle nuances of pitch at different angles of insufflation.

5 Comparison of the 24-Tone Pythagorean Model with the *Abjad* Scale

Compared to *Urmavi*'s 17-tone scale, the 24-tone Pythagorean tuning in effect in Turkish *makam* music known as *Arel-Ezgi-Uzdilek* is assembled within the octave via the affixture to the assumed tone of origin (*kaba çargah*) of 11 pure fifths upward, and 12 downward, as outlined in Table 5.

Table 5. Generation of *Arel-Ezgi-Uzdilek* by a Chain of Pure Fifths

Fifths	Frequency Ratios	Octave Normalization		Classic Interval Names
$3^{11} : 2^{11}$	177147/2048	10.	177147/131072	Pythagorean augmented third
$3^{10} : 2^{10}$	59049/1024	20.	59049/32768	Pythagorean augmented sixth
$3^{9} : 2^{9}$	19683/512	6.	19683/16384	Pythagorean augmented second
$3^{8} : 2^{8}$	6561/256	16.	6561/4096	Pythagorean augmented fifth
$3^{7} : 2^{7}$	2187/128	2.	2187/2048	apotome
$3^{6} : 2^{6}$	729/64	12.	729/512	Pythagorean tritone
$3^{5} : 2^{5}$	243/32	22.	243/128	Pythagorean major seventh
$3^{4} : 2^{4}$	81/16	8.	81/64	Pythagorean major third
$3^{3} : 2^{3}$	27/8	18.	27/16	Pythagorean major sixth
$3^{2} : 2^{2}$	9/4	4.	9/8	major whole tone
3 : 2	3/2	14.	3/2	perfect fifth
0	1/1	0.	1/1	(tone of origin – perfect prime)
2 : 3	2/3	9.	4/3	perfect fourth
$2^{2} : 3^{2}$	4/9	19.	16/9	Pythagorean minor seventh
$2^{3} : 3^{3}$	8/27	5.	32/27	Pythagorean minor third
$2^{4} : 3^{4}$	16/81	15.	128/81	Pythagorean minor sixth
$2^{5} : 3^{5}$	32/243	1.	256/243	limma, Pythagorean minor second
$2^{6} : 3^{6}$	64/729	11.	1024/729	Pythagorean diminished fifth
$2^{7} : 3^{7}$	128/2187	21.	4096/2187	Pythagorean diminished octave
$2^{8} : 3^{8}$	256/6561	7.	8192/6561	Pythagorean diminished fourth
$2^{9} : 3^{9}$	512/19683	17.	32768/19683	Pythagorean diminished seventh
$2^{10} : 3^{10}$	1024/59049	3.	65536/59049	Pythagorean diminished third
$2^{11} : 3^{11}$	2048/177147	13.	262144/177147	Pythagorean diminished sixth
$2^{12} : 3^{12}$	4096/531441	23.	1048576/531441	Pythagorean diminished ninth

The *Arel-Ezgi-Uzdilek* System (Ezgi 1933, 8-29; Özkan 2006, 45-8), with which traditional *perdes* of Turkish *makam* music are explained today, is enclosed in Table 6 below.

Table 6. *Arel-Ezgi-Uzdilek* System

Pitch	Frequency Ratios	Cents	Classic Interval Names	I. Octave *Perdes*
0:	1/1	0.000	(tone of origin – perfect prime)	*KABA ÇÂRGÂH*
1:	256/243	90.225	limma,Pythagorean minor 2nd	*Kaba Nîm Hicâz*
2:	2187/2048	113.685	apotome	*Kaba Hicâz*
3:	65536/59049	180.450	Pythagorean diminished 3rd	*Kaba Dik Hicâz*
4:	9/8	203.910	major whole tone	*YEGÂH*
5:	32/27	294.135	Pythagorean minor 3rd	*Kaba Nîm Hisâr*
6:	19683/16384	317.595	Pythagorean augmented 2nd	*Kaba Hisâr*
7:	8192/6561	384.360	Pythagorean diminished 4th	*Kaba Dik Hisâr*
8:	81/64	407.820	Pythagorean major 3rd	*HÜSEYNÎ AŞÎRÂN*
9:	4/3	498.045	perfect 4th	*ACEM AŞÎRÂN*
10:	177147/131072	521.505	Pythagorean augmented 3rd	*Dik Acem Aşîrân*
11:	1024/729	588.270	Pythagorean diminished 5th	*Irak*
12:	729/512	611.730	Pythagorean tritone	*Geveşt*
13:	262144/177147	678.495	Pythagorean diminished 6th	*Dik Geveşt*
14:	3/2	701.955	perfect 5th	*RÂST*
15:	128/81	792.180	Pythagorean minor 6th	*Nîm Zirgûle*
16:	6561/4096	815.640	Pythagorean augmented 5th	*Zirgûle*
17:	32768/19683	882.405	Pythagorean diminished 7th	*Dik Zirgûle*
18:	27/16	905.865	Pythagorean major 6th	*DÜGÂH*
19:	16/9	996.090	Pythagorean minor 7th	*Kürdî*
20:	59049/32768	1019.550	Pythagorean augmented 6th	*Dik Kürdî*
21:	4096/2187	1086.315	Pythagorean diminished 8th	*Segâh*
22:	243/128	1109.775	Pythagorean major 7th	*BÛSELİK*
23:	1048576/531441	1176.540	Pythagorean diminished 9th	*Dik Büselik*
24:	2/1	1200.000	octave	*ÇÂRGÂH*

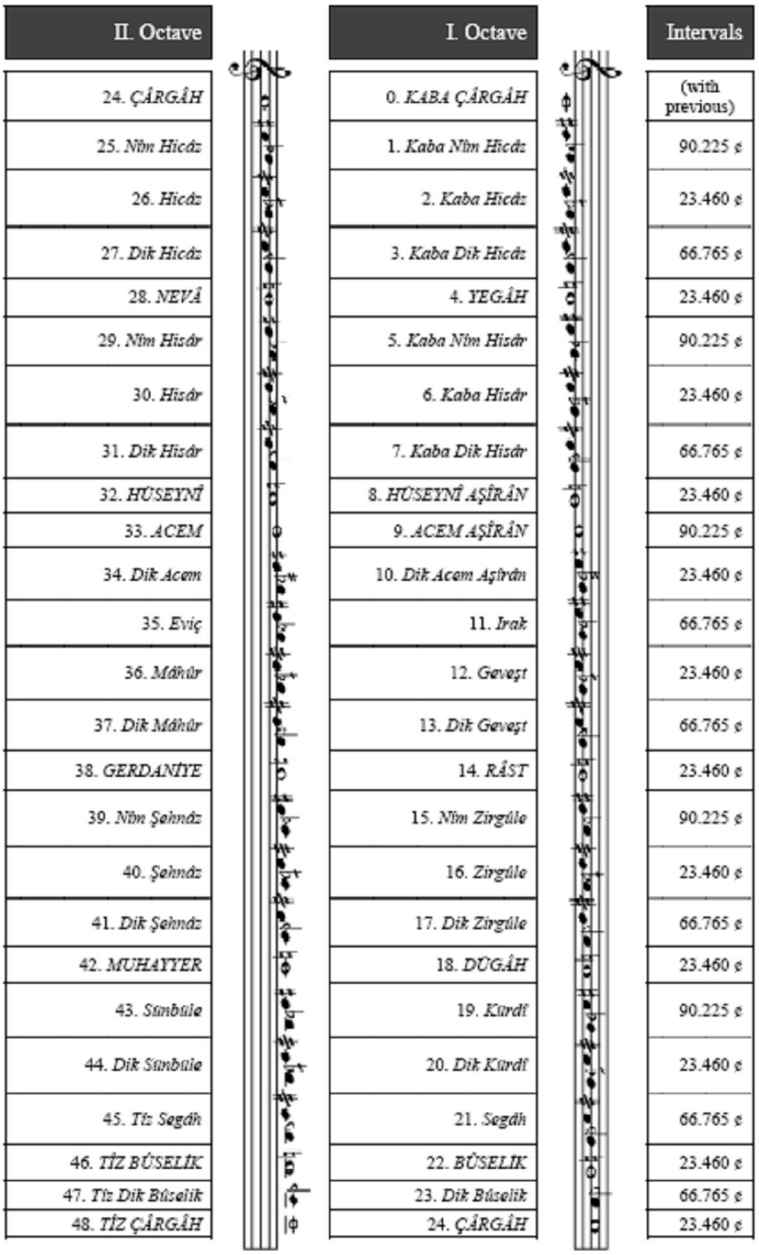
Table 6. *Arel-Ezgi-Uzdilek* System – Continued

Pitch	Frequency Ratios	Cents	Classic Interval Names	II. Octave *Perdes*
24:	2/1	1200.000	octave	*ÇÂRGÂH*
25:	512/243	1290.225	Pythagorean minor 9th	*Nîm Hicâz*
26:	2187/1024	1313.685	apotome+octave	*Hicâz*
27:	131072/59049	1380.450	Pythagorean diminished 10th	*Dik Hicâz*
28:	9/4	1403.910	major ninth	*NEVÂ* [3]
29:	64/27	1494.135	Pythagorean minor 10th	*Nîm Hisâr*
39:	19683/8192	1517.595	Pythagorean augmented 9th	*Hisâr*
31:	16384/6561	1584.360	Pythagorean diminished 11th	*Dik Hisâr*
32:	81/32	1607.820	Pythagorean major 10th	*HÜSEYNÎ*
33:	8/3	1698.045	perfect 11th	*ACEM*
34:	177147/65536	1721.505	Pythagorean augmented 10th	*Dik Acem*
35:	2048/729	1788.270	Pythagorean diminished 12th	*Eviç*
36:	729/256	1811.730	Pythagorean tritone+octave	*Mâhûr*
37:	524288/177147	1878.495	Pythagorean diminished 13th	*Dik Mâhûr*
38:	3/1	1901.955	perfect 12th	*GERDÂNİYE*
39:	256/81	1992.180	Pythagorean minor 13th	*Nîm Şehnâz*
40:	6561/2048	2015.640	Pythagorean augmented 12th	*Şehnâz*
41:	65536/19683	2082.405	Pythagorean diminished 14th	*Dik Şehnâz*
42:	27/8	2105.865	Pythagorean major 13th	*MUHAYYER*
43:	32/9	2196.090	Pythagorean minor 14th	*Sünbüle*
44:	59049/16384	2219.550	Pythagorean augmented 13th	*Dik Sünbüle*
45:	8192/2187	2286.315	Pythagorean diminished 15th	*Tîz Segâh*
46:	243/64	2309.775	Pythagorean major 14th	*TÎZ BÛSELİK*
47:	2097152/531441	2376.540	Pythagorean diminished 16th	*Tîz Dik Bûselik*
48:	4/1	2400.000	two octaves	*TÎZ ÇÂRGÂH* [4]

The habitual notation for this tuning is provided in Figure 4:

[3] Taken as 440 cps, although notated as d.

[4] Further extending until 6/1 from "*Tîz Nîm Hicâz*" to "*TÎZ GERDÂNİYE*" according to *Ezgi*.



II. Octave	I. Octave	Intervals
24. *ÇÂRGÂH*	0. *KABA ÇÂRGÂH*	(with previous)
25. *Nîm Hicâz*	1. *Kaba Nîm Hicâz*	90.225 ¢
26. *Hicâz*	2. *Kaba Hicâz*	23.460 ¢
27. *Dik Hicâz*	3. *Kaba Dik Hicâz*	66.765 ¢
28. *NEVÂ*	4. *YEGÂH*	23.460 ¢
29. *Nîm Hisâr*	5. *Kaba Nîm Hisâr*	90.225 ¢
30. *Hisâr*	6. *Kaba Hisâr*	23.460 ¢
31. *Dik Hisâr*	7. *Kaba Dik Hisâr*	66.765 ¢
32. *HÜSEYNÎ*	8. *HÜSEYNÎ AŞÎRÂN*	23.460 ¢
33. *ACEM*	9. *ACEM AŞÎRÂN*	90.225 ¢
34. *Dik Acem*	10. *Dik Acem Aşîrân*	23.460 ¢
35. *Eviç*	11. *Irak*	66.765 ¢
36. *Mâhûr*	12. *Geveşt*	23.460 ¢
37. *Dik Mâhûr*	13. *Dik Geveşt*	66.765 ¢
38. *GERDANİYE*	14. *RÂST*	23.460 ¢
39. *Nîm Şehnâz*	15. *Nîm Zirgüle*	90.225 ¢
40. *Şehnâz*	16. *Zirgüle*	23.460 ¢
41. *Dik Şehnâz*	17. *Dik Zirgüle*	66.765 ¢
42. *MUHAYYER*	18. *DÜGÂH*	23.460 ¢
43. *Sünbüle*	19. *Kürdî*	90.225 ¢
44. *Dik Sünbüle*	20. *Dik Kürdî*	23.460 ¢
45. *Tîz Segâh*	21. *Segâh*	66.765 ¢
46. *TİZ BÜSELİK*	22. *BÜSELİK*	23.460 ¢
47. *Tîz Dik Büselik*	23. *Dik Büselik*	66.765 ¢
48. *TİZ ÇÂRGÂH*	24. *ÇÂRGÂH*	23.460 ¢

Figure 4. Notation of the *Arel-Ezgi-Uzdilek* System

It is little perceived in Türkiye, that *Arel-Ezgi-Uzdilek* is actually a modification of *Rauf Yekta*'s original 24-tone Pythagorean tuning beginning on *yegah* (D) instead of the dronish and cumbersome to produce *kaba çargah* (C) (Yekta 1922, 58-9, 88-9), in which case the above-mentioned frequency ratios (hence, *perdes*) are shifted down by a major whole tone and normalized (*viz.*, reduced & sorted) within an octave – or, in other words, regenerated via the chain of 14 pure fifths down and 9 up from the new tone of origin (*yegah*), as shown in Table 7.

Table 7. Generation of *Yekta-24* by a Chain of Pure Fifths

Fifths	Frequency Ratios	Octave Normalization		Classic Interval Names
$3^9 : 2^9$	19683/512	6.	19683/16384	Pythagorean augmented second
$3^8 : 2^8$	6561/256	16.	6561/4096	Pythagorean augmented fifth
$3^7 : 2^7$	2187/128	2.	2187/2048	apotome
$3^6 : 2^6$	729/64	12.	729/512	Pythagorean tritone
$3^5 : 2^5$	243/32	22.	243/128	Pythagorean major seventh
$3^4 : 2^4$	81/16	8.	81/64	Pythagorean major third
$3^3 : 2^3$	27/8	18.	27/16	Pythagorean major sixth
$3^2 : 2^2$	9/4	4.	9/8	major whole tone
3 : 2	3/2	14.	3/2	perfect fifth
0	1/1	0.	1/1	(tone of origin – perfect prime)
2 : 3	2/3	10.	4/3	perfect fourth
$2^2 : 3^2$	4/9	20.	16/9	Pythagorean minor seventh
$2^3 : 3^3$	8/27	5.	32/27	Pythagorean minor third
$2^4 : 3^4$	16/81	15.	128/81	Pythagorean minor sixth
$2^5 : 3^5$	32/243	1.	256/243	limma, Pythagorean minor second
$2^6 : 3^6$	64/729	11.	1024/729	Pythagorean diminished fifth
$2^7 : 3^7$	128/2187	21.	4096/2187	Pythagorean diminished octave
$2^8 : 3^8$	256/6561	7.	8192/6561	Pythagorean diminished fourth
$2^9 : 3^9$	512/19683	17.	32768/19683	Pythagorean diminished seventh
$2^{10} : 3^{10}$	1024/59049	3.	65536/59049	Pythagorean diminished third
$2^{11} : 3^{11}$	2048/177147	13.	262144/177147	Pythagorean diminished sixth
$2^{12} : 3^{12}$	4096/531441	23.	1048576/531441	Pythagorean diminished ninth
$2^{13} : 3^{13}$	8192/1594323	9.	2097152/1594323	Pythagorean double dim. fifth
$2^{14} : 3^{14}$	16384/4782969	19.	8388608/4782969	Pythagorean double dim. octave

Yekta's staff notation for this 24-tone tuning – where he treats F-sharp on the 7th degree (*arak*) as F-natural (thus, turning Fb-C into a perfect fifth) at the expense and forfeiture of international legibility – is delineated in Figure 5. Following this, a comparison of *Arel-Ezgi-Uzdilek* with *Yekta-24*, and another between *Arel-Ezgi-Uzdilek* and *Abjad* Scale may be seen further in Tables 8 and 9 below. Our first comparison demonstrates the relatedness of *Arel-Ezgi-Uzdilek* to *Yekta-24*; with the only substantial difference being the "tone of origin" (*kaba çargah* vs *yegah*). The second comparison shows, that the 24-tone Pythagorean Model is none other than an extension of *Urmavi*'s 17-tone *Abjad* Scale.

II. Octave	I. Octave	Intervals
24. *NEVA*	0. *YEGÂH*	(with previous)
25. *Nim Hisar*	1. *Nim Pest Hisar*	90.225 ¢
26. *Hisar*	2. *Pest Hisar*	23.460 ¢
27. *Dik Hisar*	3. *Dik Pest Hisar*	66.765 ¢
28. *HÜSEYNİ*	4. *HÜSEYNİAŞİRAN*	23.460 ¢
29. *Acem*	5. *Acemaşiran*	90.225 ¢
30. *Dik Acem*	6. *Dik Acemaşiran*	23.460 ¢
31. *EVİÇ*	7. *ARAK*	66.765 ¢
32. *Mahur*	8. *Geveşt*	23.460 ¢
33. *Dik Mahur*	9. *Dik Geveşt*	66.765 ¢
34. *GERDANİYE*	10. *RAST*	23.460 ¢
35. *Nim Şehnaz*	11. *Nim Zengüle*	90.225 ¢
36. *Şehnaz*	12. *Zengüle*	23.460 ¢
37. *Dik Şehnaz*	13. *Dik Zengüle*	66.765 ¢
38. *MUHAYYER*	14. *DÜGÂH*	23.460 ¢
39. *Sünbüle*	15. *Kürdî*	90.225 ¢
40. *Dik Sünbüle*	16. *Dik Kürdî*	23.460 ¢
41. *TİZ SEGÂH*	17. *SEGÂH*	66.765 ¢
42. *Tiz Puselik*	18. *Puselik*	23.460 ¢
43. *Dik Tiz Puselik*	19. *Dik Puselik*	66.765 ¢
44. *TİZ ÇARGÂH*	20. *ÇARGÂH*	23.460 ¢
45. *Nim Tiz Hicaz*	21. *Nim Hicaz*	90.225 ¢
46. *Tiz Hicaz*	22. *Hicaz*	23.460 ¢
47. *Dik Tiz Hicaz*	23. *Dik Hicaz*	66.765 ¢
48. *TİZ NEVA*	24. *NEVA*	23.460 ¢

Figure 5. Notation of *Yekta-24*

Table 8. Comparison of *Arel-Ezgi-Uzdilek* & *Yekta-24*

	***AEU* Ratios**	**Cents**	***Perdes***		***Yekta-24* Ratios**	**Cents**	***Perdes***
0:	1/1	0.000	*KABA ÇÂRGÂH*				
1:	256/243	90.225	*Kaba Nîm Hicâz*				
2:	2187/2048	113.685	*Kaba Hicâz*				
3:	65536/59049	180.450	*Kaba Dik Hicâz*				
4:	9/8	203.910	*YEGÂH*	0:	1/1	0.000	*YEGÂH*
5:	32/27	294.135	*Kaba Nîm Hisâr*	1:	256/243	90.225	*Nim Pest Hisar*
6:	19683/16384	317.595	*Kaba Hisâr*	2:	2187/2048	113.685	*Pest Hisar*
7:	8192/6561	384.360	*Kaba Dik Hisâr*	3:	65536/59049	180.450	*Dik Pest Hisar*
8:	81/64	407.820	*HÜSEYNÎ AŞÎRÂN*	4:	9/8	203.910	*HÜSEYNİAŞİRAN*
9:	4/3	498.045	*ACEM AŞÎRÂN*	5:	32/27	294.135	*Acemaşiran*
10:	177147/131072	521.505	*Dik Acem Aşîrân*	6:	19683/16384	317.595	*Dik Acemaşiran*
11:	1024/729	588.270	*Irak*	7:	8192/6561	384.360	*ARAK*
12:	729/512	611.730	*Geveşt*	8:	81/64	407.820	*Geveşt*
13:	262144/177147	678.495	*Dik Geveşt*	9:	2097152/1594323	474.585	*Dik Geveşt*
14:	3/2	701.955	*RÂST*	10:	4/3	498.045	*RAST*
15:	128/81	792.180	*Nîm Zirgûle*	11:	1024/729	588.270	*Nim Zengûle*
16:	6561/4096	815.640	*Zirgûle*	12:	729/512	611.730	*Zengûle*
17:	32768/19683	882.405	*Dik Zirgûle*	13:	262144/177147	678.495	*Dik Zengûle*
18:	27/16	905.865	*DÜGÂH*	14:	3/2	701.955	*DÜGÂH*
19:	16/9	996.090	*Kürdî*	15:	128/81	792.180	*Kürdî*
20:	59049/32768	1019.550	*Dik Kürdî*	16:	6561/4096	815.640	*Dik Kürdî*
21:	4096/2187	1086.315	*Segâh*	17:	32768/19683	882.405	*SEGÂH*
22:	243/128	1109.775	*BÛSELİK*	18:	27/16	905.865	*Puselik*
23:	1048576/531441	1176.540	*Dik Bûselik*	19:	8388608/4782969	972.630	*Dik Puselik*
24:	2/1	1200.000	*ÇÂRGÂH*	20:	16/9	996.090	*ÇARGÂH*
				21:	4096/2187	1086.315	*Nim Hicaz*
				22:	243/128	1109.775	*Hicaz*
				23:	1048576/531441	1176.540	*Dik Hicaz*
				24:	2/1	1200.000	*NEVA*

Table 9. Comparison of *Arel-Ezgi-Uzdilek* & *Abjad* Scale

***AEU* Ratios**		**Cents**	***Perdes***	***Abjad* Ratios**		**Cents**	***Perdes***
0:	1/1	0.000	*KABA ÇÂRGÂH*				
1:	256/243	90.225	*Kaba Nîm Hicâz*				
2:	2187/2048	113.685	*Kaba Hicâz*				
3:	65536/59049	180.450	*Kaba Dik Hicâz*				
4:	9/8	203.910	*YEGÂH*	0:	1/1	0.000	*YEGÂH*
5:	32/27	294.135	*Kaba Nîm Hisâr*	1:	256/243	90.225	*Pest Beyati*
6:	19683/16384	317.595	*Kaba Hisâr*				
7:	8192/6561	384.360	*Kaba Dik Hisâr*	2:	65536/59049	180.450	*Pest Hisar*
8:	81/64	407.820	*HÜSEYNÎ AŞÎRÂN*	3:	9/8	203.910	*AŞİRAN*
9:	4/3	498.045	*ACEM AŞÎRÂN*	4:	32/27	294.135	*Acem Aşiran*
10:	177147/131072	521.505	*Dik Acem Aşîrân*				
11:	1024/729	588.270	*Irak*	5:	8192/6561	384.360	*ARAK*
12:	729/512	611.730	*Geveşt*	6:	81/64	407.820	*Geveşt*
13:	262144/177147	678.495	*Dik Geveşt*				
14:	3/2	701.955	*RÂST*	7:	4/3	498.045	*RAST*
15:	128/81	792.180	*Nîm Zirgûle*	8:	1024/729	588.270	*Şûri*
16:	6561/4096	815.640	*Zirgûle*				
17:	32768/19683	882.405	*Dik Zirgûle*	9:	262144/177147	678.495	*Zirgûle*
18:	27/16	905.865	*DÜGÂH*	10:	3/2	701.955	*DÜGÂH*
19:	16/9	996.090	*Kürdî*	11:	128/81	792.180	*Kürdî/Nihâvend*
20:	59049/32768	1019.550	*Dik Kürdî*				
21:	4096/2187	1086.315	*Segâh*	12:	32768/19683	882.405	*SEGÂH*
22:	243/128	1109.775	*BÛSELİK*	13:	27/16	905.865	*Bûselik*
23:	1048576/531441	1176.540	*Dik Bûselik*				
24:	2/1	1200.000	*ÇÂRGÂH*	14:	16/9	996.090	*ÇARGÂH*
				15:	4096/2187	1086.315	*Sâbâ*
				16:	1048576/531441	1176.540	*Hicâz*
				17:	2/1	1200.000	*NEVA*

6 Approximation by 53-Tone Equal Temperament

Because of the excellent proximity of either 24-tone model to the related tones of 53-equal divisions of the octave, the "9 commas per whole tone; 53 commas per octave" methodology is unanimously accepted in Turkish *makam* music parlance and education. A stereotypical schema pertaining to the *Arel-Ezgi-Uzdilek* division of the whole tone is reproduced in Figure 6 (Özkan 2006: 46).

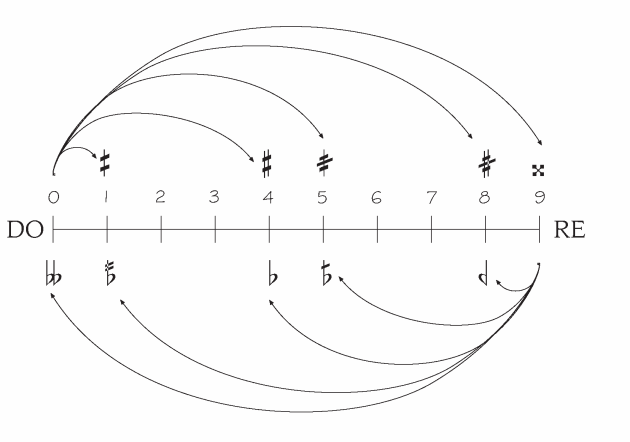


Figure 6. *Arel-Ezgi-Uzdilek* Division of the Whole Tone into 9 commas [5]

The frequency ratios of and intervals between these accidentals – including their counterparts in *Yekta-24* and equivalents in 53 equal divisions of the octave – are projected onto Table 10.

[5] Depiction reproduced from p. 46 of the reference to this figure. The correct range, however, should have been Fa-Sol. Each comma is Holdrian, *i.e.*, ~22.642 cents wide, hence, one step of 53 equal divisions of the octave – which is a decent approximation with less than a cent error to the Pythagorean comma (difference of a stack of 12 pure fifths from 7 octaves) expressed as $3^{12} : 2^{19} = 531441:524288$ and equalling 23.46 cents.

Table 10. Exposition of the Fa-Sol Division in *Arel-Ezgi-Uzdilek* & *Yekta-24*

	***AEU* Ratios**	**Notation**		***Yekta-24* Ratios**	**Notation**		**Intervals**	**53-tET, Cents**
0:	4/3	F	G 𝄫	32/27	F		(with previous)	(22.-31.)
1:	177147/131072	F 𝄰	G𝄬	19683/16384	F 𝄰	G𝄬	531441:524288	22.642 ¢
2:								
3:								
4:	1024/729	F ♯	G 𝄬	8192/6561	F ♯	G♭	134217728:129140163	67.925 ¢
5:	729/512	F 𝄯	G𝄬	81/64	F 𝄪	G 𝄬	531441:524288	22.642 ¢
6:								
7:								
8:	262144/177147	F 𝄪	G𝄮	2097152/1594323	F 𝄪	G 𝄭	134217728:129140163	67.925 ¢
9:	3/2	F𝄪	G	4/3		G	531441:524288	22.642 ¢

How well 53-tone equal temperament embodies both *Arel-Ezgi-Uzdilek* and *Yekta-24* (and therefore the *Abjad* Scale) to the point of doing away with either may be seen in Table 11.

Table 11. Approximation of *Arel-Ezgi-Uzdilek* & *Yekta-24* by 53-tET

***AEU* Ratios (*Abjad* Scale in bold)**		**Cents**	***Yekta-24* Ratios (*Abjad* Scale in bold)**		**Cents**	**53-tET Aprx.**	**Diff.**
0:	**1/1**	0.000	0:	**1/1**	0.000	**0:** 0.000	0
1:	**256/243**	90.225	1:	**256/243**	90.225	**4:** 90.566	0.341
2:	2187/2048	113.685	2:	2187/2048	113.685	**5:** 113.208	-0.4775
3:	**65536/59049**	180.450	3:	**65536/59049**	180.450	**8:** 181.132	0.6821
4:	**9/8**	203.910	4:	**9/8**	203.910	**9:** 203.774	-0.1364
5:	**32/27**	294.135	5:	**32/27**	294.135	**13:** 294.340	0.2046
6:	19683/16384	317.595	6:	19683/16384	317.595	**14:** 316.981	-0.6139
7:	**8192/6561**	384.360	7:	**8192/6561**	384.360	**17:** 384.906	0.5457
8:	**81/64**	407.820	8:	**81/64**	407.820	**18:** 407.547	-0.2728
9:	**4/3**	498.045	9:	2097152/1594323	474.585	**21:** 475.472	0.8867
			10:	**4/3**	498.045	**22:** 498.113	0.0682
10:	177147/131072	521.505				**23:** 520.755	-0.7503
11:	**1024/729**	588.270	11:	**1024/729**	588.270	**26:** 588.679	0.4093
12:	729/512	611.730	12:	729/512	611.730	**27:** 611.321	-0.4093
13:	**262144/177147**	678.495	13:	**262144/177147**	678.495	**30:** 679.245	0.7503
14:	**3/2**	701.955	14:	**3/2**	701.955	**31:** 701.887	-0.0682
15:	**128/81**	792.180	15:	**128/81**	792.180	**35:** 792.453	0.2728
16:	6561/4096	815.640	16:	6561/4096	815.640	**36:** 815.094	-0.5457
17:	**32768/19683**	882.405	17:	**32768/19683**	882.405	**39:** 883.019	0.6139
18:	**27/16**	905.865	18:	**27/16**	905.865	**40:** 905.660	-0.2046
19:	**16/9**	996.090	19:	8388608/4782969	972.630	**43:** 973.585	0.9549
			20:	**16/9**	996.090	**44:** 996.226	0.1364
20:	59049/32768	1019.550				**45:**1018.868	-0.6821
21:	**4096/2187**	1086.315	21:	**4096/2187**	1086.315	**48:**1086.792	0.4775
22:	243/128	1109.775	22:	243/128	1109.775	**49:**1109.434	-0.341
23:	**1048576/531441**	1176.540	23:	**1048576/531441**	1176.540	**52:**1177.358	0.8185
24:	**2/1**	1200.000	24:	**2/1**	1200.000	**53:**1200.000	0
(Average absolute difference: 0.4486 cents, Highest absolute difference: 0.9549 cents)							

7 Conclusions

In this article, we have reviewed *Al-Kindi*'s ud fretting and *Abjad* notation, and have comparatively evaluated, seemingly for the first time, two historical notations in Turkish *makam* music based on the *Abjad* numerical system. Of particular interest is *Urmavi*'s 17-tone *Abjad* Scale spanning two octaves whose precursor is *Al-Kindi*'s tuning. Upon it, *Nasir Dede* ascribed traditional *perde* names recognized today.

However, the handful of extant musical examples written in *Abjad* prove its lack of popularity among composers and executants throughout the ages. As expected, *Abjad* is no longer in use today.

Next, we compared the theory in effect in Türkiye known as *Arel-Ezgi-Uzdilek* with *Yekta-24* and showed that both were essentially the same, and showed that the 24-tone Pythagorean Model was simply an extension of the 17-tone *Abjad* Scale.

Finally, we demonstrated that 53 equal divisions of the octave was a common grid embracing the said tunings with less than a cent error.

On close scrutiny, a gross asymmetry in the deployment of *Arel-Ezgi-Uzdilek* accidentals catches the eye, and F# (4 commas sharp) not being the same distance from F as Gb (5 commas flat) is from G, to say nothing of Fx and Gbb not being double at all, leaves something to be desired.

In retrospect, *Yekta*'s symbols may be found to be less disproportionate by comparison – particularly if the Fa-Sol region is notated properly as shown in Table 10.

Even so, *Yekta-24* is handicapped due to diatonic naturals not being the product of an uninterrupted cycle of fifths [6], a feature *Arel-Ezgi-Uzdilek* flaunts despite its lack of credentials for a Pythagorean C-major scale running from *kaba çargah* to *çargah* as the basis of Turkish Music theory (Levendoğlu 2003: 181-93; Aksoy 2003: 174-5).

Yekta-24 is further dysfunctional, in that the order of sharps and flats in the chain is not faithful to Western idiom. *Arel-Ezgi-Uzdilek* is likewise encumbered in the sharps sector.

It is not surprising, therefore, that the resources of 53-tone equal temperament, particularly in regard to transpositions and polyphony, are not fully utilized in Turkish *makam* music. Hence, 53 equal divisions of the octave – far from being wholly implemented on any acoustic instrument of *makam* music – serves rather theoretical interests, especially when delineating customary melodic inflexions during practice.

[6] Since, in the series C-G-D-A-E-B-F#, the interval between E-B (262144:177147) is a wolf fifth of 678.5 cents, and B-F# is found at the other end of the chain 8-9 fifths below C.

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