Novel Theory Leads to the Classical Outcome for the Precession of The Perihelion of a Planet due to Gravity

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Abstract: We offer a novel method which lets us derive the same classical result for the precession of the perihelion of a planet due to the gravitational effects of the host star. The theoretical approach suggested earlier by the first author is erected upon just the energy conservation law, which consequently yields the weak equivalence principle. The precession outcome is exactly the same as that formulated by the General Theory of Relativity (GTR) for Mercurial orbit eccentricities, but the methodology used is totally different. In our approach, there is no need to make any categorical distinction between *luminal* and *sub-luminal matter,* since, as we have previously demonstrated, our theory of gravity is fully compatible with the foundations of quantum mechanics. Our approach can immediately be generalized to the many-body problem, which is otherwise practically impossible within the framework of GTR. Our approach thus leads to a unified description of the micro and macro world physics.

Key Words: Gravitation, General Relativity, Yarman's Approach, Precession, Law of energy conservation

Résumé: Nous offrons une nouvelle formulation en vue de prédire la précession classique du périhélie d'une planète, dans le champs gravitationnel d'une étoile. Cette approche originalement proposée par le premier auteur, est basée uniquement sur la loi de conservation de l'énergie, se donnant au principe d'équivalence gravitationnelle faible, où la masse au repos de l'objet en question, disparait de l'équation de mouvement. Notre résultat de précession est le même que celui qui est produit par la théorie de la relativité générale (TRG), pour une planète parcourant une orbite elliptique de faible eccentricité, bien que les philosophies derrière les deux théories, sont totalement différentes, l'une de l'autre. Il est important de noter que dans notre approche, il n'est pas nécessaire de faire distinction entre le photon et la matière ordinaire, comme nous l'avons démontré dans un précédent travail consacré à l'étude de la déflection de la lumière; cette propriété rend notre théorie de gravitation tout à fait compatible avec la mécanique ondulatoire. Notez que fondamentalement notre approche peut facilement être généralisée au problème à plusieurs corps (ce qui est pratiquement impossible dans le cadre de la TRG). Notre approche permet en outre une unification facile de nos descriptions du monde micro et du monde macro.

I. INTRODUCTION

In 2013, we published a paper about our new cosmological model erected upon the law of energy conservation, which consequently yields the *weak equivalence principle* [1]. In that work, we were able to predict Hubble's Law, and at the same time, provide an answer to the *dark energy quest*. According to our approach, what is widely referred to as *"dark energy"* turned out to be the *residue* of a very small but positive acceleration of about $10^{-9}\ g$ (where g represents Earth's surface acceleration) of the initial protracted expansion of a multi-layer cosmic egg structure, whose radius can be maximally compressed to no further than roughly 2 billion light years. We also showed that, at some earlier epochs of our universe's expansion, the acceleration might have been negative throughout (*i.e.* deceleration), which remains in good harmony with what is being conjectured today.

In a subsequent paper [2] once more based on our novel theory, we elaborated on the fact that the bending caused by gravitation of visible light all the way down to radiowaves happen to be exactly equal to what is predicted by the General Theory of Relativity (GTR). On the other hand, we surmised that electromagnetic rays specifically bear a photonic kernel of rest mass [3]. Our approach insured, at any case, the "same gravitational potential" as that of GTR, which yields under the same framework the usual light bending, Shapiro delay, and the precession of the perihelion of the orbit of Mercurial planets.

In this article, we tackle the precession of the perihelion of a planet in a gravitational field based upon the novel theory presented by the first author with the support of his colleagues [4-8], which we hereforward refer to as Yarman's Approach (YA). According to this theory, the gravitational field energy may anyway be a non-vanishing quantity in all possibly definable frames of reference. The application of the law of energy conservation entails not only that we land at the similitude of *gravitational and inertial masses,* but it also means that, the proper mass of the object at hand is cancelled out of the resulting equation of motion. This property is readily recognizable as the *weak equivalence principle.*

In our approach, the proper mass $m_{0\infty}$ is the mass measured at infinitely far away from everything else, where we suppose the effect of gravity on it thoroughly vanishes. Said mass of an object, when it is brought into and embedded inside a gravitational field, is altered in such a way that, its *overall motion-wise relativistic energy E,* were it furthermore in motion as referred to by a distant observer outside of their influence, can be described by the simple relationship [4,5]

$$E = \gamma m_{0\infty} c^2 \left(1 - E_B / m_{0\infty} c^2\right) \; , \tag{1}$$

where γ is the habitual Lorentz factor associated with the motion of the object, *c the velocity of light in empty space,* and E_B is the *static binding energy; i.e.* the energy required in order to bring the object of rest mass $m_{0\infty}$ conceptually weighed at a distance of infinity (thus free of the influence of any field) quasistatically (as if gently pulled by a "rope") back to infinity away. One can anyway think of this object as sitting at rest on a celestial body, where we also suppose that the host mass is stationary in space, so as to avoid dealing with the problem of a celestial body in rotation around itself.

The reader should be cautioned that, the static binding energy E_B is *not* the *"total binding energy"* of the object revolving around a given star, which would evidently be smaller than the *static binding energy* due to the kinetic energy associated with the motion of the object. The *total binding energy* is the energy necessary to furnish to the object in its trajectory around a given star to take it out of orbit and carry it to infinitely far away. Conversely, the static binding energy is the energy one would have to furnish to the object at rest (*i.e.* suspended) at a given location in the gravitational field to bring it quasistatically to a distance of infinity.

In a closed system, the total energy E as expressed in eq. (1) must naturally remain *constant* owing to the law of energy conservation. Thus, objects with different rest masses indeed acquire the same acceleration in a given gravitational field. Note that, this latter occurrence holds valid not just in Albert Einstein's equation of motion, but also within the framework of Isaac Newton's equation of motion. It is just as well valid in our approach.

The likeness of gravitational and inertial masses always allows the choosing of a reference frame where the local geometry intrinsically becomes pseudo-Euclidean. In such a frame, a body continues to experience the presence of the gravitational field owing to and commensurate with the variance in its rest mass compared to what it would have weighed in the total absence of gravity.

As shown previously [9-11], the *decrease of the rest mass* of the given object induces a corresponding change of its temporal and spatial units, which we call respectively $T_{empty\ space}$ and $L_{empty\ space}$, indicating a virtual *"original measurement"* of said object's properties taken in totally empty space. These quantities become T and L when the object is embedded in a gravitational field as assessed by a distant observer outside the interacting system, which then occur to be a function of the static gravitational binding energy, *i.e.* [8]

$$T = \frac{T_{empty\ space}}{1 - E_B / m_{0\infty} c^2}, \quad L = \frac{L_{empty\ space}}{1 - E_B / m_{0\infty} c^2} \ . \tag{2a}$$

The static binding energy is proportional to $m_{0\infty}$; thus the above equations do not at all depend on knowing what $m_{0\infty}$ is.

Eq. (2a) fosters a metric that is *conformally flat,* along with a non-constant conformal factor. We also observe that eq. (2a) is not subjected to any restrictions when extending the local geometry uniformly to the entire space-time. Although, from an operational viewpoint, we can only speak about a local geometry in a given location for a given object, unless information on the properties of other objects located at other spatial points is a priori available [1].

The above equations are indeed very easy to work with. Supposing we propose to measure the distance of a tidally locked and circularly revolving planet around the Sun to this star, this distance, say r_0, can be measured by an observer situated on the given planet via him sending a light-beam of velocity c from the planet to the Sun, and detecting the bounce-back from the Sun, and finally working out the local period of time t_0 the beam has taken to go forth and come back. The distance r_0 hence becomes equal to ct_0. Meanwhile, the locally measured period of time t_0 turns out to be shorter than the corresponding period of time t computed by a distant observer practically unbound to any gravitational field [cf. Eq.(2a)]. The same distance for him, which we now call r, takes the value of ct.

Since, as we will show in due course, the speed of light in vacuum remains always constant in YA, the elapsed period of time the inhabitant of the planet established locally the way we described constitutes a (local) measurement of the distance r_0. The remote observer though, virtually unassociated with the solar system we are dealing with, will assess it as r owing to

$$r = tc = \frac{t_0 c}{1 - E_B / mc^2} = \frac{r_0}{1 - E_B / mc^2} \ , \tag{2b}$$

because his clock runs faster in comparison, and that, as much as delineated by the first equation of the set of eqs. (2a).

Or, the other way around, since the local clock on the planet runs slower compared to the distant observer's clock in empty space, the *number of ticks* the inhabitant of the planet would record for the round trip of the signal he sent to the Sun will be less than the tick count the outside observer would tally for the same round trip inferred using an identical method and related triangulation.[a] This entails that, r_0 assessed by the local observer is in effect shorter than r when assessed by the distant observer as much as the static binding energy coming into play as formulated in equation (2b).

The infinitesimal change in the altitude of the object at hand in the gravitational field of concern induces a change in its static binding energy E_B commensurate with a variance in its rest mass, and ergo, the subsequent change of metric coefficients through eqs. (2a) when applied to the co-moving reference frame of this body. Such modification of metric in the co-moving frame versus r is *sensed* (so to speak) by the object as the *"gravitational force"* [1].

In the framework of YA, the overall change of the metric of space-time in the frame co-moving with this object represents rather a secondary effect caused by the transformation of its rest mass under gravity in concord with a change of its static binding energy. We interpret this to preclude any imposition by a space-time curvature based on classically pre-inserted metric recipes [1].

The essential predictions of GTR (such as gravitational red shift, precession of the perihelion of Mercury, bending of visible light) are classically considered to be the direct results of *curved space-time* due to the presence of a sizeable mass. In our approach, though, they become fundamental and intrinsic quantum mechanical effects based on the deep-seated law of energy conservation.

Our approach also puts into doubt the rather abstract concept of *"field"*. *"Force"*, in contrast, is a tangible *change* effectuated by a ponderable source on a proximous object, and is not really the result of the interaction of this latter object by an *"altered environment",*

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[a] The distant observer O can well calculate the distance r between the Sun (S) and the planet (P) in question via sending a light beam to S as well as another to P. The round trip of the first takes the period of time t_{OS}, and that of the second t_{OP}. This procedure yields respectively the distances $OS=ct_{OS}$ and $OP=ct_{OP}$. By measuring the angle β for the sides OS and OP, he can then figure out $r=PS$ based on the known triangle relationship

$$PS^2 = r^2 = c^2 t_{OS}^2 + c^2 t_{OP}^2 - 2\, c^2 t_{OS} t_{OP} cos\beta \ ,$$

yielding therefore,

$$r = c\sqrt{t_{OS}^2 + t_{OP}^2 - 2\, t_{OS} t_{OP} cos\beta} \ .$$

Once the distant observer has reached this outcome, he can then obtain r_0 as the distance assessed by the local inhabitant of the planet to the Sun via applying eq. (2b).

hence, the so called *"field"*. The change, according to YA, presumably occurs at the inside of the body itself [10-12].

Gravitational red shift too has been considered under the framework of YA in Refs. [4,5,8] as a simple and clearly understandable quantum mechanical effect. Interested readers may further consult our papers [12-14].

The light bending result we have arrived at betokens that a photon may bear a non-vanishing rest mass. In this regard, we do not make any distinction between a luminal and a ordinary sub-luminal object [2]. The propagation velocity v of a photon with a kernel of rest mass is thusly conjectured to be just a bit less than the uppermost theoretical limit c, and a convergence towards c depends, in general, on its frequency. In other words, the higher the frequency of the photon at hand, the closer its speed v is to c. However, the assumed difference between v and c could remain, to all intents and purposes, indistinguishable even when using the most advanced technology and experimental techniques of quotidian science [15]. At any case we leave aside the light bending problem in this paper.

The established general relativistic derivation of the precession of the perihelion of a planet can be found in any related textbook. It is also offered in Einstein's book [16] next to the light bending calculations. Einstein based his latter calculations therein on *Fermat's Principle* [17], which is demonstrably improper as elucidated in the upcoming footnote.

In section 2, we reproduce the derivation of the precession of the perihelion of a planet starting from an energy conservation relationship obtained within the framework of GTR. Next, in section 3, we reproduce the elemental aspects of YA basically from Ref. [5]. Then, in section 4, we tackle the problem of the alleged gravitational constant, which really is not a universal constant according to YA, but increases with the strength of gravity. In what follows in section 5, we present our general equation of motion yielding the precession of the perihelion of a planet.[b] Next, in section 6, we provide a simple and concise calculation of the precession of the perihelion of the orbit of a planet. We leave the presentation of the rigorous orbital calculations and so forth to a subsequent work; since, this is not really our aim in the present article. Finally, we draw a conclusion in section 7.

The precession results happen to be astonishingly the same in both GTR and YA for chiefly orbits featuring small eccentricities such as that of Mercury, despite the fact that there exists insurmountable incompatibilities between the two theories. Future measurements of higher orbit eccentricities may offer a way of ascertaining the validity of which approach under consideration comes closer to reality.

II. PRECESSION OF THE PERIHELION OF A PLANET AS PER GTR REVISITED

Consider a relatively big mass $\mathcal{M}$, such as that of the Sun, as the source of a "gravitational field". The Schwarzschild metric in isotropic form is given in [18,19]. Here, we make use of a two-dimensional formulation expressed by

$$ds^2 = (1-2\alpha)c^2dt^2 - (1+2\alpha)(r^2d\theta^2 + dr^2) \ , \tag{3}$$

where

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[b] A. Einstein originally offered in his book [16] a derivation of light bending based on Fermat's Principle, where he concluded light velocity in GTR should decrease nearby the ponderable mass under consideration, which consequently induces a related bending. T. Yarman chose for simplicity to do the same in his original derivation of light bending, and also for that of the precession of the perihelion of Mercury [5] to obtain the deviation of the trajectories from the Newtonian trajectories, thereby producing his own results. Yet, the application of Fermat's Principle, while amazingly producing the expected results, turns out to be inappropriate. More specifically, while light is slowing down nearby the ponderable mass of concern, say, along a vertical direction from $z=-\infty$ to $z=0$ (the latter being the impact point; and the trajectory of light is assumed, for simplicity's sake, to be a straight line), one obtains via applying the Fermat principle a deflection angle along the horizontal direction toward the gravity source amounting to $\vartheta/2$ (*i.e.* half of the expected total deflection angle). However, at the spatial domain corresponding to the variation of z from 0 to $+\infty$, the velocity of light is now increasing with z according to GTR (no matter how awkward this may sound, seeing as such is contrary to common sense to imagine an object having grazed a celestial body speed up after it goes past it, given that one would contrariwise expect it to slow down due to the tug of gravity), and the application of the Fermat Principle yields a deflection angle of $\vartheta/2$, still in the horizontal, but in the opposite direction of that of the first one (thus away from the source of gravity). In other words, the overall deflection angle, if we choose to use the Fermat Principle the way Einstein did, must come out to be zero! So, the proposed setup based on Fermat's Principle is improper. One wonders, though, howcome the algebraic setup based on Fermat's Principle still leads to the expected results. To keep this story short, we will leave related explanation aside. Suffice it to say that, any setup based on the Fermat Principle is conceptually invalid no matter how fortuitously correct results may be reproduced through its employment.

$$\alpha = G_0\mathcal{M}/rc^2 , \tag{4}$$

and where r, θ are polar coordinates, G_0 is the habitual gravitational constant, and c is the velocity of light in vacuum. Note that, all of the above coordinates are those measured by the distant observer practically outside of the influence of said gravitational environment.

Within the framework of the metric (3), we see that lengths are contracted and periods of time stretched as referred to by the distant observer in comparison to what they would have been at a location in empty space free of the gravitational effects of the host star. Thus, an infinitely short *(proper)* spatial length dl_L lying at a given altitude from the center of $\mathcal{M}$ as determined by a fixed observer situated at this altitude will be measured to be dl when measured by the distant observer, so that

$$dl = dl_L\sqrt{1-2\alpha} . \tag{5}$$

The time periods are dilated under gravitation in GTR. That is to say, the locally measured *(proper)* period of time $d\tau$ associated with a clock situated at the given altitude is determined to be dt when assessed by the distant observer, so that

$$dt = d\tau/\sqrt{1-2\alpha} . \tag{6}$$

Now, consider an object in motion nearby $\mathcal{M}$ crossing the piece of trajectory dl in orbit during the period of time dt, as assessed by the distant observer. The coordinate velocity v (*i.e.* the one observed by the distant observer) is by definition

$$v = dl/dt . \tag{7}$$

The fixed local observer, instead, will measure the velocity v_L to be,

$$v_L = dl_L/d\tau . \tag{8}$$

The two velocities relate to each other via eqs. (5) and (6):

$$v = \frac{dl}{dt} = \frac{dl_L\sqrt{1-2\alpha}}{d\tau\sqrt{1+2\alpha}} \approx v_L(1-2\alpha) , \tag{9}$$

with the accuracy of calculations leaning towards c^{-2}.

This means, *proper velocities* in GTR decrease by the factor $(1-2\alpha)$ under gravitation as gauged by the distant observer. Recall that, this holds for any velocity, and therefore too for the velocity of light. The *proper velocity* of light c_0 in GTR*,* as ascertained at empty space free of any gravitational field, decreases nearby the mass $\mathcal{M}$ when assessed by a distant observer at an altitude r to become c*,* so that

$$c = c_0(1-2\alpha). \tag{10}$$

This is the purported explanation of the well-known *Shapiro delay* [20]. Take heed though, while Einstein's application of the Fermat Principle to the light bending phenomenon did turn out to be improper as explained in the previous footnote, the metric sketched at the level of eq. (3) leads to the expected bending angle of light passing a nearby star.

Consider at this point, when we say *"speed of light",* we mean throughout this article c_0*,* and we shall denote it from now on simply as c, and not as c_0.

We hereby propose to illustrate how the precession of the perihelion of concern, based on GTR, can be derived starting from an expression written for the *total relativistic energy* [21],

$$E = \gamma m_{0\infty} c^2 \sqrt{g_{00}} \quad , \tag{11}$$

when an object with a rest mass $m_{0\infty}$ *(established at infinitely far away from all else)* is moving in a static (or stationary) gravitational field. Here g_{00} is the time-time component of the *general relativistic* metric tensor, $\gamma = \left(1 - v_L{}^2/c^2\right)^{-1/2}$ is the usual Lorentz factor of the given object, and the subscript "$_L$" means that the velocity v_L is ascertained by a *local observer* resting at a given location on the orbital trajectory of the planet.

In the chosen metric, eq. (1) takes the form of

$$E = \gamma m_{0\infty} c^2 \sqrt{1-2\alpha} \ , \tag{12}$$

which says that the *relativistic rest energy* $m_{0\infty}c^2$ of an object, when brought into the vicinity of a gravity source, is decreased according to the GTR by the coefficient $\sqrt{1-2\alpha}$ by virtue of the space-time curvature induced by the presence of the host mass.[c] This is the *general relativistic gravitational red shift effect* coming into play for an object embedded in a gravitational field.

On top of that, the *relativistic rest energy* is increased as much as the Lorentz factor as drawn from the Special Theory of Relativity (STR) when the object at hand is brought to a motion around the host. In such a framework, eq. (12) is easy to grasp; it is the *overall general relativistic energy* of the planet moving within the gravitational field of the star.

Next, we suppose that the rest mass $\mathcal{M}$ of the host object is so much heavier than the rest mass $m_{0\infty}$ of the orbiting object so as to render it inert, *i.e.* $\mathcal{M} >> m_{0\infty}$. In this limit, the energy expressed by eq. (12) in GTR represents a given constant value denoted by *C*,

$$\gamma m_{0\infty}c^2\sqrt{1-2\alpha}=C\,. \tag{13}$$

Taking the square of this

$$\frac{1-2\alpha}{1-\frac{v_L{}^2}{c^2}}=\frac{C^2}{\left(m_{0\infty}c^2\right)^2}\,, \tag{14}$$

then differentiating it, gives us

$$d\alpha\left(1-\frac{v_L{}^2}{c^2}\right)=(1-2\alpha)\frac{v_L dv_L}{c^2}\,. \tag{15}$$

Via the definitions made in eq. (4), and eq. (5), one then arrives at

$$-\frac{G\mathcal{M}}{r^2}\left(1-\frac{v_L{}^2}{c^2}\right)dr_L\sqrt{1-2\alpha}=(1-2\alpha)v_L dv_L\,,\ \text{or} \tag{16}$$

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[c] Note that, rest energies are decreased, while masses are increased due to the assumption of GTR on the equivalence of the effects of acceleration and gravitation. Whereas in YA, both rest energies and masses decrease by the same factor in a gravitational field.

$$-\frac{G\mathcal{M}}{r^2}\left(1-\frac{{v_L}^2}{c^2}\right)dr_L = \sqrt{1-2\alpha}\,v_L dv_L \quad . \tag{17}$$

Recall the common binomial expansion for a very small α as compared to unity:

$$\sqrt{1-2\alpha} \cong 1-\alpha \quad . \tag{18a}$$

Moreover, for a nearly circular orbit, which arguably can be said to be the case of Mercury, we can approximately write

$$\frac{G\mathcal{M}}{r^2} \approx \frac{v_L^2}{r}, \text{ or} \tag{18b}$$

$$\frac{G\mathcal{M}}{rc^2} = \alpha \approx \frac{v_L^2}{c^2} \quad . \tag{18c}$$

Thereby - *for a nearly circular orbit* - eq. (17) becomes

$$-\frac{G\mathcal{M}}{r^2}dr_L = v_L dv_L \quad . \tag{19a}$$

At the LHS of this equation we now have Newton's gravitational "field intensity" $G\mathcal{M}/r^2$. For a relatively small α, eq. (19a) can be more appropriately re-written as [cf. eqs. (17) and (18a)]

$$-\frac{G\mathcal{M}}{r^2}\left(1-\frac{{v_L}^2}{c^2}\right)(1+\alpha)dr_L = dv_L \quad . \tag{19b}$$

On the other hand, the *Newtonian vector attraction force* exerted by $\mathcal{M}$ on a unit mass of the planet situated at a distance *r,* as well as the local vector acceleration $d\mathbf{v}_L/d\tau$ of the planet, lie along the same direction, but face the opposite direction than the radial direction $\boldsymbol{r}$ (which we consider as directed outward).

We now take into account the local velocity vector [eq. (8)],

$$\mathbf{v}_L = d\boldsymbol{r}_L/d\tau \quad , \tag{19c}$$

where the vectors $\mathbf{v}_L$ and $d\boldsymbol{r}_L$ are parallel to each other.

Hence,

$$\frac{\mathbf{r}_L \cdot d\mathbf{r}_L}{r_L dr_L} = \frac{\mathbf{v}_L \cdot d\mathbf{v}_L}{v_L dv_L} \text{ , and} \tag{20}$$

$$dr_L = \frac{\mathbf{r}_L \cdot d\mathbf{r}_L}{r_L} \frac{v_L dv_L}{\mathbf{v}_L \cdot d\mathbf{v}_L} \quad . \tag{21}$$

Substituting this equality into eq. (19a), we get

$$-\frac{G\mathcal{M}}{r^2} \frac{\mathbf{r}_L \cdot d\mathbf{r}_L}{r_L} = \mathbf{v}_L \cdot d\mathbf{v}_L \quad , \tag{22a}$$

and dividing this outcome by $d\tau$ on both sides, which is the differential period of time of the local observer, we obtain

$$-\frac{GM}{r^2} \frac{r_L \cdot v_L}{r_L} = v_L \cdot \frac{dv_L}{d\tau} \quad . \tag{22b}$$

Through the cancellation of $\boldsymbol{v}_L$ terms, one finally gets to the vectorial equation of motion:

$$-\frac{G\mathcal{M}}{r^2} \frac{\mathbf{r}}{r} = \frac{d\mathbf{v}_L}{d\tau} \quad . \tag{23a}$$

This is interesting, for it says that *GTR straightforwardly furnishes, on the local level, the Newtonian Equation of motion for a planet travelling in a nearly circular orbit* (something not widely known at all).

Using eq. (9), we can, still for a nearly circular orbit (for which α practically stays constant), write in vector form

$$d\mathbf{v} = d\mathbf{v}_L (1-2\alpha) \text{ , or} \tag{23b}$$

$$\mathrm{d}\mathbf{v}_L = \mathrm{d}\mathbf{v} /(1-2\alpha) . \tag{23c}$$

Furthermore, using eq. (8), we can express eq. (23a) via the coordinate velocities and accelerations (as measured by a distant observer):

$$-\frac{G\mathcal{M}}{r^2} \frac{\mathbf{r}}{r} = \frac{d\mathbf{v}}{(1-2\alpha)^{3/2}\, dt} \text{ , or} \tag{23d}$$

$$-\frac{G\mathcal{M}}{r^2} (1-3\alpha) \frac{\mathbf{r}}{r} = \frac{d\mathbf{v}}{dt} \ , \tag{24}$$

where we have taken into account the common expansion

$$\left(1-2\alpha\right)^{3/2} \approx 1-3\alpha \ , \tag{25}$$

in the adopted accuracy of our calculations.

For nearly circular orbits, we can practically write, via eq. (18b)

$$-\frac{G\mathcal{M}}{r^2}\left(1-3\frac{v^2}{c^2}\right)\frac{\boldsymbol{r}}{r}=\frac{d\boldsymbol{v}}{dt} \ , \tag{26}$$

where, on the LHS, we can legitimately rely on the approximation

$$v_L \cong v \, . \tag{27}$$

Thus in GTR, it is the term in between the parentheses on the LHS of eq. (26) which makes the perihelion of the planet to precess around a star.

Remembering how we arrived at the above results, one can summarize in the proceeding steps that the precession in GTR is due to i) the motion of the object, ii) the contraction of lengths, and iii) the stretching of the units of the period of time. The last two transformations are conjointly responsible for the decrease of the local velocity when assessed by the distant observer. Eq. (26) is useful for comparing the effects due to GTR to those due to YA with respect to the precession of the perihelion of the orbit of a planet.

The precession of the perihelion in YA on the other hand, is, as shall be revealed further on, attributable to i) the motion of the object (just like it is the case in GTR), ii) the quantum mechanical stretching of the sizes commensurate with a decrease of the rest mass of the bound object, and iii) the related dilation of time. The last two occurrences taking place together secure the constancy of the velocity of light (or any other speed) under gravitation in YA.

We will further see that, the new scalings of masses, lengths and periods of time coming into play under the umbrella of YA entail that the gravitational constant, is *not really a universal constant after all*, but increases with the strength of the gravitational field. Notice furthermore, that YA, unlike GTR, does not embody any singularities (infinitely dense point-size objects). This surely has profound implications for cosmology if ascertained.

In the next section, we will show that YA predicts practically the same precession outcome when the eccentricity of the elliptic orbit is small (*e.g.* when the orbit is nearly circular). Instead of using the perturbational calculation to determine the divergence of the new orbit from the classical orbit (which Yarman originally undertook in Ref. [5]), we will henceforward provide a direct and concise calculation of the precession of the perihelion.

III. ELEMENTAL ASPECTS OF YARMAN'S APPROACH FOR GRAVITATIONAL INTERACTION

Suppose one has under consideration a universe consisting of just two celestial bodies gravitationally interacting with each other, such as our Sun and Mercury. Let $\mathcal{M}$ be the mass of the much heavier Sun, and $m_{0\infty}$ the mass of the planet when initially weighed at infinity away, where all the gravitational force on it vanishes.

The constraint $\mathcal{M} >> m_{0\infty}$ we just framed for the masses above is indeed not a necessity for the approach we will momentarily sketch. We do so that, when $m_{0\infty}$ is in motion around $\mathcal{M}$, this latter always stays in place as viewed from the perspective of a distant observer virtually outside of the influence of their interaction and at rest with the planet, so that we do not have to deal with a two-body scenario for the moment, which can still be solved anyway according to YA as shown in [22].

We will heed to the following basic procedure for conceiving the motion of Mercury around the Sun based upon YA:

First, we shall visualize bringing Mercury *quasistatically* from infinitely far away to a chosen radius r on its prospective orbit around the Sun, and hold it suspended there for a moment (*i.e.* at rest). Second, we shall imagine to deliver to it the impetus to kick it into its orbital motion.

The first step, owing to the law of energy conservation and broadened to include the mass and energy equivalence of the Special Theory of Relativity (STR), yields in YA a decrease in the rest mass of $m_{0\infty}$ as much as the *static binding energy* $B(r)$ coming into play [4,5]. Thus, $m_{0\infty}$ becomes $m(r)$, so that

$$m(r)c^2 = m_{0\infty}c^2 - B(r)\,. \tag{28}$$

Let us precise that c is the speed of light in empty space. We have thus applied a principal law of YA as elucidated right below.

Law 1: The rest mass (or the same, *rest energy,* were the speed of light taken unity) of an object bound to a celestial host body, in fact to any plausible source of gravitation or other force field it may interact with – *such as the case of, say, two hydrogen atoms bound to an oxygen atom in a water molecule, or a muon, for instance, bound to an atomic nucleus* – amounts to less than the object's rest mass measured in "empty space"; and this, as much as its "static binding energy" vis-à-vis the attraction source of concern.

If one further carries mass $m(r)$ quasistatically away from $\mathcal{M}$ as much as dr, he has to furnish energy against the gravitational pull $\mathcal{M}$ exerts on $m(r)$; which then, owing to Law 1, will yield an increase in the mass $m(r)$ as much as $dm(r)$. Therefore we may, in our familiar notation, write:

$$dm(r)c^2 = G_0\frac{\mathcal{M}\,m(r)}{r^2}dr\,, \tag{29}$$

where G_0 is the known "Universal Gravitational Constant".

We need to attract the attention of the reader to the fact that G_0 is not a *"universal constant"* in YA. Indeed, when embedded inside a gravitational field, as we shall soon see in the next section, the so-called gravitational constant transforms to become G as assessed by the distant observer. Here the *"Gravitational Force"* is nothing else but the usual *Newtonian Attraction Force,* but defined our way.

If the masses $m(r)$ and $\mathcal{M}$ of concern are not at rest with respect to each other, then, just as Newton himself suspected, the law of gravitation between these two masses is not anymore governed by the expression $G_0 m(r)\mathcal{M}/r^2$.

This latter expression is for when referred to by the local observer as shall be illustrated below [7]. Note further, we do not really need to borrow the law of gravitational attraction from Newton [5], since we have demonstrated that it is in fact, a requirement imposed by the STR; though, strictly for masses at rest [5].

The integration of eq. (29) yields immediately the decreased mass $m(r)$:

$$m(r) = m_{0\infty} e^{-\alpha} , \tag{30}$$

where α is the same as demarcated by eq. (4).

The comparison of eqs. (28) and (30) bequeaths the *static binding energy* $B(r)$ at the given location r:

$$B(r) = m_{0\infty} c^2 \left(1 - e^{-\alpha}\right) . \tag{31}$$

Along the course of the mentioned *second step,* we now envision hurtling Mercury into orbit around the Sun from its suspended altitude r, to where we had already brought the planet quasistatically from infinitely far away to a position at rest. Mercury will hence bear the velocity v in its *orbital trajectory* of concern at the given location.

This yields the *Lorentz increase* of the *rest mass* $m(r)$ at r, so that the *overall relativistic mass* m_γ, or the same, the *overall relativistic energy* of the object (which for an isolated system ought to remain *constant* throughout in orbit), becomes [see eq. (1)]

$$m_\gamma c^2 = m_{0\infty}c^2 \frac{1-\dfrac{B(r)}{m_{0\infty}c^2}}{\sqrt{1-\dfrac{v^2}{c^2}}} = m_{0\infty}c^2 \frac{e^{-\alpha}}{\sqrt{1-\dfrac{v^2}{c^2}}} = Constant \quad . \tag{32}$$

This equation encapsulates the *overall relativistic energy of the object on the given orbit under the framework of YA*. The above is, in effect, the elaborate form of eq. (1) re-written based on the present approach.

There is a striking similarity with this equation of the total relativistic energy obtained under the framework of GTR [see eqs. (12), (13) and Ref. 22]. They yield the same result up to a third order Taylor Expansion, and naturally both produce the Newtonian equation of motion as a first approximation. This is quite extraordinary, and has been highlighted in a previous work [cf. Ref. 5].

In YA, the *instantaneous orbital velocity v* is an invariant both for the distant observer and the local observer on the planet [5]. Accordingly, the speed of light (or in fact any speed) remains the same under gravitation in YA when measured by either the local observer, or the distant observer.

This property is not at all an assumption, but arises from the quantum mechanical aspects of the theory. Distances and periods of time under gravitation are transformed in the same direction owing to inherent quantum mechanical inter-play of quantities when a rest mass change is introduced into the quantum mechanical description of the object at hand [9-11]. The rest mass of the object under consideration decreases via eq. (31) through attraction in any field it comes to interact with, thus yielding quantum mechanically a corresponding size increase and a stretching of the period of time by exactly the same amount as measured by a distant observer.[d] The resultant clock retardation in YA is due to the *slowing down of the*

[d] And this is precisely why the speed of light, according to the present approach, remains unchanged when passing near and around a celestial body; while bearing in mind the fact that light will still take a longer amount of time to graze the body of concern as referred to by the distant observer, but now due simply to the increasing of the distance alone – which thence remarkably accounts for the Shapiro delay, but through a totally different philosophy.

internal dynamics of the object, thereby causing a gravitational red shift. Whereas, in GTR, distances are contrariwise contracted and periods of time are stretched [see eqs. (2a) and (2b)].

Observe also, mass in GTR (owing to the principle of equivalence as originally advanced by this theory between the effect of acceleration and the effect of gravitation) increases under the gravitational field, whereas *energy* decreases; which creates serious difficulties with respect to the thorough implementation of the law of energy conservation in GTR [23]. In YA, however, both mass and energy vary in the same direction under gravity.

At this point, it might be useful to summarize how mass, size, time period and energy deviate in a gravitational field according to both GTR and YA. Table 1 reflects the coefficients that are to be multiplied with the original quantities of concern, that are ideally measured in empty space free of any field and when embedded in a gravitational medium with respect to a remote observer.

Table 1 How mass, size, period of time and energy at rest vary in a gravitational field in GTR and YA?

	Mass	Size	Period of time	Energy	Velocity of light
GTR	$1/\sqrt{1-2\alpha}$	$\sqrt{1-2\alpha}$	$1/\sqrt{1-2\alpha}$	$\sqrt{1-2\alpha}$	$1-2\alpha$
YA	$e^{-\alpha}$	e^{α}	e^{α}	$e^{-\alpha}$	Invariant
Explanation	About the same but in opposite directions	About the same but in opposite directions	About the same	About the same	Gets changed in GTR, but remains constant in YA

IV. EQUATION OF MOTION AND THE VARIATION OF THE GRAVITATIONAL CONSTANT WITH THE INTENSITY OF THE FIELD IN YARMAN'S APPROACH

Eq. (32), via differentiation, leads to

$$-\frac{G_0 \mathcal{M}}{r^2 c^2}\left(1-\frac{v^2}{c^2}\right)dr = d\alpha\left(1-\frac{v^2}{c^2}\right) = \frac{vdv}{c^2}\ . \tag{33}$$

This equation is valid for any object in a given trajectory within a closed system because of its general construction, since it does not make any *distinction* between a light particle bearing a kernel of rest mass (no matter how tiny this may be) and ordinary sub-luminal matter [5]. In this sense, the photon is conceived to be an ordinary object moving at a speed less than the theoretical barrier *c,* no matter how close *v* may be to *c*. Hence, the greater the energy of such a photon, the closer its speed is to the uppermost ceiling velocity *c.* (Anyway as conveyed, the interaction of specifically photons with a celestial body, is left aside in this article.)

At any rate, in YA, the so-called gravitational constant is not a universal invariant at all. Accordingly, eq. (33) is to be slightly modified, as shall be discussed below.

Let us recall that, eq. (33) is written in the frame of the local observer as has been drawn from the fabric of eq. (29). Nonetheless, converting it into vectorial form, we obtain

$$-\frac{G_0\,\mathcal{M}}{r^2}\left(1-\frac{v^2}{c^2}\right)\frac{\boldsymbol{r}}{r}=\frac{d\,\mathbf{v}}{dt}\ . \qquad (34)$$

Here, $\boldsymbol{r}$ is the position vector issuing from the center of the Sun and pointing to the given instantaneous location of either the photon or any other sub-luminal object.

So far, we have tacitly assumed that the gravitational constant G_0 is not affected by the source of gravity. Yet, in both GTR and YA, the gravitational constant is *dimension-wise altered* in a gravitational field. Surely, a quantity the overall exponent of whose dimensional scaling coefficients does not vanish cannot remain constant in the gravitational medium of concern.

It is troubling that this fact has been overlooked in GTR since near a century. Recall that, the assumed gravitational constant bears the dimensions of force×size^2/mass^2, or (mass×size×size^2)/(time^2×mass^2). Table 2 clarifies how the alleged gravitational constant varies dimensionally under a gravitational field for both GTR and YA.

Table 2 How the assumed "gravitational constant" varies dimensionally in a gravitational field for GTR and YA)*

	How does the alleged gravitational constant vary?	Overall coefficient coming into play
GTR	(p) mass×(1/p) size×(1/p^2) size^2/[(p^2) time^2×(p^2) mass^2]	1/p^6
YA	(1/p) mass×(p) size×(p^2) size^2/[(p^2) time^2×(1/p^2) mass^2]	p^2

* The coefficient p is $1/\sqrt{1-2\alpha}$ for GTR, and e^{α} for YA.
For relatively small α's, both quantities are practically equal to each other.

Thanks to this dimensional analysis, we can easily observe that G_0 remains constant in neither GTR, nor in YA. Thus, it is incorrect to consider it to be a universal measure at any rate! Nevertheless, in YA, just like in STR, not only *c,* but Planck's Constant *h,* as well as e^2 (the square of an electric charge in Gaussian units) remain fully constant.[e]

In any case, let us return to YA and ask: How does one measure the gravitational constant? The answer lies with the famous Cavendish Experiment [24]. To determine the *proper gravitational constant* G_0 in the laboratory, the *proper force* f_0 exerted by a ball of a given proper mass m_0 on another ball of the same proper mass m_0 suspended from one edge of a balance that is counter-weighted at the other end with another suspended proper mass m_0 needs to be assessed. The distance between the two interacting balls of mass m_0 each is $\mathcal{R}_0$. In accordance with our adoption of Newton Force as entered into Eq.(29) above, the gravitational constant G_0 then becomes

[e] The fourth co-author triggers our awareness with regards to the fact that, electron charge intensity *e* expressed in *StatCoulombs* (otherwise known as *StatC*, or the same, *esu* under the CGS unit system) instead of *e* expressed in *Coulombs* under the MKS unit system is Lorentz Invariant. That is, electrostatic units are not altered when brought to a uniform translational motion, whereas Coulombs are. It directly implies the Lorentz Invariance of the quantity e^2/ε_0 in the MKS unit system (where ε_0 is the customary term for the permittivity of free space) instead of just e^2 in Coulombs^2. This can be drawn from the dimensionless Fine Structure Constant $\alpha = \frac{e^2}{2\varepsilon_0 hc}$, where ε_0 is identifiable as $\frac{10^7\,\mathrm{C}^2}{4\pi|c^2{}_{\mathrm{MKS}}|\mathrm{Nm}^2} = 8.85418782 \times 10^{-12}$ Farads / m using the relationships $\varepsilon_0\mu_0 = 1/c^2$ (μ_0 being the magnetic permeability of vacuum) and $\mu_0 = 4\pi\mathrm{x}10^{-7}$ Henrys per meter (being fixed by definition), with $|c_{MKS}|$ indicating the modulus of the velocity of light in meters per second. Given that *hc* is a Lorentz Invariant quantity, its complement e^2/ε_0 (*i.e.* Newton force x Surface Area in squaremeters) occurring in the formula for the Fine Structure Constant ought also to be a Lorentz Invariant quantity to properly cancel the dimensions. Finally, the unusual transformation relationship between the charge intensity expressed in CGS and MKS unit systems can be presented in the form of e_{StatC} = 10 x $e_{Coulombs}$ x $|c_{\mathrm{MKS}}|$, which is something not widely recognized.

$$G_0 = \frac{f_0 \mathcal{R}_0^2}{m_0^2} \ . \tag{35}$$

However, the *distant observer* in YA will, according to Table 2, require to transform eq. (35) to arrive at his G (with regards to the Cavendish equipment embedded in Earth's gravitational field) as follows:

$$G = G_0 e^{2\alpha} \ . \tag{36}$$

The small distance $\mathcal{R}_0$ between the Cavendish balls becomes $\mathcal{R} = \mathcal{R}_0 \, e^{\alpha}$ when assessed by the distant observer [see eq. (2b)]. We therefore conclude that, it is actually $G_0/\mathcal{R}_0^2$ which remains invariant in YA.[f] Consequently,

$$\frac{G}{G_0} = \frac{\mathcal{R}^2}{\mathcal{R}_0^2} \ . \tag{37}$$

How, under these circumstances, will the distant observer express the gravitational force F between, say, the Sun and the Earth (or the Sun and Mercury), when both the star and the planet are assumed to be at rest with respect to each other? The answer, based on the discussion which ensued so far, is none other than

$$F = G\frac{\mathcal{M}\, m(r)}{r^2} \ . \tag{38}$$

All of the quantitites appearing here are those assessed by the distant observer. $\mathcal{M}$ is, as conveyed previously, the mass of the Sun, $m(r)$ is the mass of the planet at a distance r from the Sun, and G is defined by eq. (36). The distant observer, on the other hand, would have his own gravitational constant G_0 after having conducted his own Cavendish experiment at a

[f] The invariance of the quantity G_0/R_0^2 is fascinatingly echoed at the atomistic level in the demonstrable proportionality $e^6/\hbar^4$, where e is the electric charge in *StatCoulombs* (hence Lorentz Invariant, unlike *Coulombs*), with the denominator containing the fourth power of the Reduced Planck Constant; or otherwise, in the identical MKS relationship $e^6\pi/h^4\,4\varepsilon_0^{\,3}$ (where ε_0 is the electrical permittivity of vacuum). Note that, anything Lorentz Invariant remains unchanged in YA whether or not it is embedded inside gravity (or any other type of field it can interact with).

place practically free of any surrounding gravitational influence – *i.e.,* on a celestial abode of negligle mass using small Cavendish balls.

In effect, what one measures on Earth, pretty much satisfies the ideal conditions, because Earth is a relatively light celestial body in comparison to the Sun, or many other stars. One can similarly well or even better define the proper gravitational constant G_0 on a tiny asteroid practically free of any gravitational field.

It is in any case important to note that the proper quantity G_0 or in fact any other quantity, locally measured anywhere, i.e. either in almost empty space, or in a strong gravitational environment, shall in YA, turn out to be the same, just like a stick meter locally speaking is everywhere the same stick meter (provided that it preserves its "identity") . The thing is, if a local observer assesses the given quantiy, but located at a different gravitational altitude than his, then he would come out with a different number for this quantity, say G_0. Thence, although all local observers would *locally* come with the same number for G_0, there is no symmetry between obervers as to, they would assess from their own locations, each other's G_0 differently, as detailed above.

Let us continue anyway: A local observer on Earth can measure his proper distance r_0 to the Sun by sending a light signal, receiving it back, and counting the ticks in the interim on his own clock. This is the distance r for the remote observer. The relationship between r and r_0 , in YA, using eqs. (2a) and (2b) next to Table 1, is

$$r = r_0 e^{\alpha} \ . \tag{39}$$

We have thus defined r_0 as the distance measured by an observer on the planet, who sends to the Sun a light beam, detects the beam bounced back from the Sun, and counts the number of ticks his local clock would display at the end of the beam's journey. Since the velocity of light traversing empty space in YA is under all conditions a *universal constant,* the number of ticks the local observer would register by watching his local clock constitutes a local estimation of the distance of the planet to the star of concern. But since the clock of the remote observer ticks faster, the distance r will be determined to be longer than r_0 owing to the necessary transformation as explained in the Introduction.

We ought to stress again, that the velocity of light is anyway a universal constant in YA, and, unlike in GTR, stays always constant on its way to the star and back to the planet. This makes the mathematical description of the present approach much easier to set up than the one advanced by GTR.

Note further that, eq. (39) is an intrinsically quantum mechanical representation of the scenario at hand (see Refs. 10-12). If the rest mass of an object in its quantum mechanical description is lessened, then its total energy will as well. When the rest mass is decreased under the gravitational field, its energy must decrease in the same way. As a consequence, its period of time conjointly stretches just as much, meaning the slowing down of the local clock, while its size proportionally lengthens as specified in eq. (39). This is nothing else but the classical gravitational red shift, this time due not to space-time curvature, but simply by virtue of quantum mechanical transformations. As a result, YA is fully compatible with quantum mechanics, and, in fact, opens a novel way to unify it with relativity the way we attempt here.

Eq. (39) implies that we can rewrite eq. (37) in the following form:

$$\frac{G}{r^2} = \frac{G_0}{r_0^2}. \tag{40}$$

Via eq. (38), the force between the Sun and the Earth becomes

$$F = G_0 \frac{\mathcal{M}\, m(r_0)}{r_0^2}, \tag{41}$$

as assessed by the remote observer, but now in terms of a gravitational constant value pinned down in practically empty space, while the distance of the planet to the Sun has been measured locally as explained.

Next to the ratio G/r^2, another invariant in YA appears to be the quantity *gravitational constant* × *mass*^{2} *or force* × *distance*^{2} (cf. Table 1). This means that, the Newtonian gravitational attraction force $G_0 \mathcal{M} m(r_0)/r^2$ is the force measured with respect to the local observer on the planet.

The notion of the gravitational attraction force (for masses strictly at rest) as assessed by the remote observer designating r_0, while the same force as assessed by the local observer designating r instead may seem awkward. But it must be the way it is, if one wants to use the gravitational constant G_0 measured at ideally empty space practically free of gravitational surroundings in the given force expressions. A discussion of this issue is provided in Ref. [7].

Before we close this section, let us briefly check if the gravitational force we employed in eq. (29), as measured by the local observer attached to the planet, and the identical force, but as assessed by the distant observer expressed through eq. (38), are well compatible within the present framework.

We can notice right away, that the ratio of [the local force] / [the identical force measured by the distant observer], amounts to $(r_0^2/r^2) = e^{-2\alpha}$ [(see eq. (39)]. On the other hand, we can consider that the force has the dimensions of mass×velocity/period of time. Masses in YA, when embedded in gravity, are affected by $e^{-\alpha}$ when assessed by a distant observer (see Table 1), and periods of time are conjointly stretched by e^{α}; velocities though, are untouched. Therefore, the local force bears an intensity $e^{-2\alpha}$ smaller than that of the identical force as assessed by the distant observer. The cross-check thence provides the expected outcome.

V. THE PRECESSION OF THE PERIHELION OF A PLANET ACCORDING TO YARMAN'S APPROACH

Equation (33) is relevant for a local observer. In order to express it with respect to the distant observer, we have to replace r by r_0 via using eq. (39) Thus, eq. (33) can be re-written as

$$-\frac{G_0 \mathcal{M}}{r_0^2}\frac{e^{-\alpha}}{1+\alpha}\left(1-\frac{v^2}{c^2}\right)dr_0 = v_0 dv_0 \quad . \tag{42}$$

Recall that, the gravitational force F as assessed by the distant observer between the star and the planet, both of which are at rest to him, is given in eq. (41). So, in the LHS of the above equation, we specifically have the gravitational attraction force $G_0 \mathcal{M}/r_0^2$ exerted by the source mass $\mathcal{M}$ on a unit test mass at rest as viewed by the distant observer.

In eq. (42), we alternatively wrote $v_0 dv_0$ instead of vdv, since the velocities in YA are left unaltered at any rate. Now, we divide both sides by the infinitely short local period of time term dt_0 to obtain [5]

$$-\frac{G_0 \mathcal{M}}{r_0^2}\frac{e^{-\alpha}}{1+\alpha}\left(1-\frac{v^2}{c^2}\right)\frac{dr_0}{dt_0} = \frac{v_0 dv_0}{dt_0} \quad \text{, or} \tag{43}$$

$$-\frac{G_0 \mathcal{M}}{r_0^2}\frac{e^{-\alpha}}{1+\alpha}\left(1-\frac{v^2}{c^2}\right) = \frac{dv_0}{dt_0} \quad . \tag{44}$$

Using the technique followed at the level of eqs. (20) and (21), we can transform the latter equation into the corresponding vector equation:

$$-\frac{G_0 \mathcal{M}}{r_0^2}\frac{e^{-\alpha}}{1+\alpha}\left(1-\frac{v^2}{c^2}\right)\frac{\mathbf{r}}{r} = \frac{d\,\mathbf{v}_0}{dt_0} \quad . \tag{45}$$

We can discern that one is entitled to write

$$\frac{d\,\mathbf{v}}{dt} \cong \frac{d\,\mathbf{v}_0}{dt_0} \quad ; \tag{46}$$

that is, the accelerations measured both locally and by the remote observer are about the same.

The reason for, is that, dt is practically equal to dt_0. Indeed, from eq. (39) one can obtain

$$dr = \frac{dr_0\, e^{\alpha}}{1+\alpha} \cong dr_0 \; . \tag{47}$$

Since lengths and periods of time stretch just as much in the present approach, one can similarly obtain

$$dt = \frac{dt_0\, e^{\alpha}}{1+\alpha} \cong dt_0 \; . \tag{48}$$

The difference between eq. (45) and eq. (42) is thus insignificant, even for relatively strong fields for which α is around *1/2*.

Notice that, we could just as well proceed from the vector eq. (34) onward via using eq. (39) to attain

$$-\frac{G_0\,\mathcal{M}}{r_0^2} e^{-2\alpha}\left(1-\frac{v^2}{c^2}\right)\frac{\mathbf{r}}{r} = \frac{d\mathbf{v}}{dt} \; ; \tag{49}$$

which is, owing to eq. (46), practically the same as equation eq. (45).

Eq. (45), or its similar, eq. (49) as we disclosed above, is the general equation of motion as assessed by the distant observer, irrespective of the distance r_0 between the Sun and the planet measured by the local observer on the planet. It provides both the precession of the perihelion of the orbit, and the light bending for photons having a kernel of rest mass; thus making it unnecessary to distinguish, on the whole, luminal and ordinary sub-luminal objects.

Note that, for small α, eq. (45) immediately reduces to eq. (34). Moreover, eq. (42) is the same equation as eq. (26) obtained using a GTR setup for the case of a planet in a nearly circular orbit around the Sun.

So far, we have demonstrated that GTR and YA produce the same outcome to all intents and purposes, but through totally different theoretical foundations. While eq. (42) is rigorous, the analogous equation obtained in GTR is valid only for nearly circular orbits. We summarize in Table 3 the reasons for why a planet's perihelion precesses in its orbit around the Sun according to GTR and YA respectively. As surprising as it may seem, virtually the same amount of precession is predicted by both theories. The calculation of the rigorous orbit is left for a subsequent work.

Table 3 Elements making a planetary orbit precess around the Sun according to GTR and YA

	Why does a planet exhibit precession of the perihelion in its orbit? It is due to the following reasons.	*Explanations*
GTR	i) The motion of the object. ii) The contraction of lengths. iii) The stretching of units period of time.	The last two items are responsible for the slowing down of the local velocity as assessed by the *distant observer.* The same goes for the speed of light. The *gravitational constant* is kept unaltered in GTR, whereas it should vary.
YA	i) The motion of the object. ii) The quantum mechanical stretching of lengths, which go along with the mass decrease. iii) The conjoint quantum mechanical stretching of periods of time.	The last two items together secure the constancy of the velocity of light under a gravitational field. The scaling of masses, lengths and periods of time entail that *G* is not a universal constant, but increases with the strength of the gravitational field.

VI. CALCULATION OF THE PRECESSION OF THE PERIHELION BASED ON YARMAN'S APPROACH

To compare the predictions of GTR and YA apropos the precession of the perihelion, let us start by assuming a nearly circular orbit. Newtonian Equation of Motion (18-b) dictates

$$\frac{G_0\,\mathcal{M}}{r_N} = v^2 \ , \tag{50}$$

where r_N is the orbit's radius predicted by the Newtonian approach with regards to the given velocity v.

On the other hand, eq. (26) delineated by GTR, or similarly by the corresponding YA eq. (42), for small α and nearly circular orbits predict the orbital radius r for a given velocity v for both GTR and YA:

$$\begin{aligned}&\frac{G_0\,\mathcal{M}_0}{r^2}\left(1-3\frac{v^2}{c^2}\right)\cong\frac{G_0\,\mathcal{M}_0}{r^2}\left(1-\frac{v^2}{c^2}\right)\mathrm{e}^{-2\alpha}\\&\cong\frac{G_0\,\mathcal{M}_0}{r^2}\left(1-\frac{v^2}{c^2}\right)(1-2\alpha)\cong\frac{v^2}{r}\end{aligned}\quad . \tag{51a}$$

For a nearly circular orbit this yields

$$\frac{G_0\,\mathcal{M}_0}{r}(1-3\alpha)\cong v^2 \quad . \tag{51b}$$

The angular velocity ω_N of the planet for the given velocity v in the Newtonian approach is, by definition,

$$\omega_N = \frac{v}{r_N} \quad . \tag{52}$$

In contrast, the angular velocity ω of the planet as assessed by a distant observer in either GTR or YA for the *given velocity v* is, again by definition,

$$\omega = \frac{v}{r} \quad , \tag{53}$$

with r being the instantaneous distance of the planet to the Sun corresponding to v.

The angular velocity ω of the planet, as predicted by GTR or by YA, can therefore be expressed in terms of the Newtonian angular velocity ω_N :

$$\omega = \omega_N \frac{r_N}{r} = \frac{\omega_N}{1-3\alpha} \quad . \tag{54a}$$

Using eqs. (50) and (51b) to evaluate the magnitude of the ratio r_N/r, we finally get

$$\omega = \frac{\omega_N}{1-3\alpha} \quad . \tag{54b}$$

This equation shows that, the angular velocity is a bit larger in either GTR or YA than predicted by the Newtonian gravity setup. During one Newtonian revolution of period T, the orbit's perihelion according to GTR or YA (yet, for small α, and nearly circular orbits) will be advanced by

$$\vartheta_{Precession} = (\omega - \omega_N)T \cong 3\alpha\omega_N T \cong 3\frac{G_0\mathcal{M}}{rc^2}\frac{v}{r}T = 3\frac{G_0\mathcal{M}}{rc^2}\frac{2\pi r}{rT}T = 6\pi\frac{G_0\mathcal{M}}{rc^2} \ . \tag{55}$$

Thus, YA produces exactly the same result as predicted by GTR for a nearly circular orbit.

Let us go further ahead by considering an elliptical orbit. The approximation[g] [cf. the side by side division of eqs. (50) and (51b)]

$$\frac{r_N}{r} \approx \frac{1}{1-3\alpha} \quad , \tag{56}$$

holds satisfactorily even in this case, and we can still use it in the LHS of eq. (54a). One can thus rewrite eq. (55), but this time as an integration over the Newtonian elliptical orbit:

$$\vartheta_{Precession} = \oint(\omega - \omega_N)dt \cong \oint 3\alpha\omega_N dt \quad . \tag{57}$$

The angular momentum ω_N is, by definition,

$$\omega_N = \frac{d\varphi_N}{dt} \quad , \tag{58}$$

where φ_N is the *polar angle* centered at the focus of the ellipse hosting the Sun.

The quantity α was defined in eq. (4), and the polar coordinate *r* is given by

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[g] Concerning a Newtonian elliptical orbit of semi-major axis a_N at a distance r_N from the Sun, where the planet has the measured velocity *v,* one can write

$$G_0\,\mathcal{M}_0\left(\frac{2}{r_N} - \frac{1}{a_N}\right) = v^2 \ .$$

For the present case involving an open elliptical orbit of semi-major axis *a* at the corresponding distance *r* from the Sun, yet where the planet still has the measured velocity *v,* the above relationship turns into

$$G_0\,\mathcal{M}_0\left(\frac{2}{r} - \frac{1}{a}\right)(1-3\alpha) = v^2 \ .$$

We would now like to evaluate r_N/r which, for chiefly small eccentricities, can be written as a_N/a at the perihelion. Therefore,

$$\frac{r_N}{r} = \frac{\frac{1}{r}}{\frac{1}{r_N}} = \frac{\frac{2}{r}}{\frac{2}{r_N}} \cong \frac{a_N}{a} = \frac{\frac{1}{a}}{\frac{1}{a_N}} \cong \frac{\frac{2}{r} - \frac{1}{a}}{\frac{2}{r_N} - \frac{1}{a_N}} \ .$$

To arrive at the above equalization, we made use of a well-known property, which is that, if a ratio (third one from the left) is equal to another one (fifth one from the left), then we can define a new ratio (at the very right hand side) consisting of a numerator made up of the difference of their numerators and a denominator made up of the difference of their denominators, the result being equal to the original ratio. Finally, via dividing the second orbital equation at the top with the first one at the top, we attain precisely the ratio r_N/r:

$$\frac{r_N}{r} \cong \frac{\frac{2}{r} - \frac{1}{a}}{\frac{2}{r_N} - \frac{1}{a_N}} = \frac{1}{1-3\alpha} \quad (qed) \ .$$

$$r = \frac{p}{1+\varepsilon\cos\varphi} \quad , \tag{59}$$

where ε is the *eccentricity* of the elliptical orbit, and p is given by

$$p = \frac{b^2}{a} , \tag{60}$$

with a and b being the semi-major and semi-minor axes, respectively, of the ellipse.

Under these circumstances, eq. (57) becomes

$$\vartheta_{Precession} \cong \oint 3\frac{G_0\,\mathcal{M}}{rc^2}\frac{d\,\varphi_N}{dt}dt = \oint 3\frac{G_0\,\mathcal{M}}{c^2}\frac{\left(1+\varepsilon cos\,\varphi\right)}{p}d\,\varphi_N \quad . \tag{61}$$

This readily leads to

$$\vartheta_{Precession} = 6\,\pi\frac{G_0\,\mathcal{M}\,a}{b^2c^2} \quad , \tag{62}$$

which is exactly what GTR predicts for higher orbit eccentricities.

Interestingly enough, if we had used $v^2/\,c^2$ instead of α in a rigorous derivation, the result would still be the same [25]. Recall that α is of the order of 10^{-8} for Mercury. Therefore, the approximation of $e^{-2\alpha}$ by $1-2\alpha$ is very well justified. A visible difference may come into play near very compact bodies for which α becomes much larger, and for which one may need to use one more term in the Taylor expansion of $e^{-2\alpha}$. The resulting difference can be calculated without difficulty, but this will not be considered in this paper.

VII. CONCLUSION

Under the framework of Yarman's Approach (abbreviated as YA), we presented a concise derivation of the precession of the perihelion of a Mercurial planet, and compared it to what is, in contrast, yielded by a setup based on GTR.

Both approaches herein were founded on the conservation of energy in a closed system. Conceptual structure due to GTR was wholly preserved in our formulations. We discovered that, the same precession outcome is obtained in both GTR or YA for Mercurial or even higher orbit eccentricities, even though the two theories are built on fundamentally different assumptions embodying conflicting properties. This, we find amazing. Future measurements of higher orbit eccentricities may offer a way of ascertaining the validity of which approach under consideration comes closer to reality.

Table 3 summarizes the conceptual differences between GTR and YA.

In GTR, the precession of the perihelion of a planet is due to i) the motion of the planet, ii) the contraction of lengths, and iii) the stretching of the units of the period of time. The last two items are conjointly responsible for the decrease of the local velocity when assessed by the distant observer.

In YA, the precession phenomenon is due to i) the motion of the planet just like it is the case with GTR, ii) the quantum mechanical stretching of the lengths commensurate with a decrease due to the binding of the object's rest mass to the host mass, and iii) the resultant quantum mechanical stretching of the periods of time, causing the so-called "gravitational constant" to vary depending on the strength of gravity. Furthermore, the last two items together secure the constancy of the velocity of light (or any other speed) under gravitation in empty space.

While it is evident today that there are insurmountable difficulties for unifying GTR with quantum mechanics, these difficulties vanish when applying the YA framework as shown here. YA is fundamentally erected on just the law of energy conservation. This consequently yields the weak equivalence principle. Our approach thus leads to a unified description of the micro and macro world physics. It can furthermore be generalized to elegantly address many-body scenarios, which even today remain a daunting task to set up in GTR.

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