# —CLARIFICATION OF "OVERALL RELATIVISTIC ENERGY" ACCORDING TO YARMAN'S APPROACH—

Ozan Yarman[1], Metin Arık[2], Ricardo Gobato[3], Tolga Yarman[4]
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Abstract. In this essay, we will attempt to clarify the concept of "overall relativistic energy" according to *Yarman's Approach*; which happens to be the underlying framework of *Yarman-Arık-Kholmetskii* (YARK) gravitation theory. The reformed meaning of this key concept is, in juxtaposition to the general theory of relativity (GTR), shown to subtly differ from particularly the Newtonian understanding of the "total energy of a system" as just being the "sum of constituent kinetic and potential energies".

*Keywords*: Relativistic energy, Total gravitational energy, YARK Theory, General Relativity, Classical Mechanics, Newtonian Mechanics.

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## 1. Introduction

At the onset of our previous contribution [1], we chronicled the progress of the team led by the fourth co-author from the foundation of the *Universal Matter Architecture* (UMA) scaffolding [2, 3, 4, 5, 6, 7, 8] to the establishment of *Yarman's Approach* for all force interactions [9, 10], which eventually led to the development of *Yarman-Arık-Kholmetskii* (YARK) gravitation theory [11, 13, 14, 15, 16, 17, 18, 19] (where a symbiosis between Quantum Mechanics and gravitation was harmoniously achieved — with the associated gravitational field energy becoming a non-vanishing quantity in all possibly definable reference frames). Let us now take a step back to visit one particularly re-occuring key concept of our framework: A system's *total or overall relativistic energy.*

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[1] Prof. Dr., İstanbul University, İstanbul, Turkey.
e-mail: ozanyarman@gmail.com;
[2] Prof. Dr., Boğaziçi University, İstanbul Turkey.
[3] Prof. Dr., Laboratory of Biophysics and Molecular Modeling Genesis, Parana, Brazil.
[4] Prof. Dr., Okan University, İstanbul Turkey.

In many of the publications cited above, what the authors refer to as the "overall relativistic energy" or "total relativistic energy" keeps coming up. This, in effect, takes place when explaining the atomistic world or the celestial world either within the context of *Yarman's Approach* or the more generalized YARK theory of gravity. Recall that the methodology of the fourth co-author and his colleagues do not differentiate between the microcosm and the macrocosm from the ground level all the way to the heavens. For the contribution at hand, we would especially like to clarify what is meant by "overall (viz., total) relativistic energy", and exemplify it in a simple two-body gravitational scenario. To reiterate, we use the designation "overall energy" synonymously with the denomination "total energy" under the framework at hand.

Before all else, it is useful to remember that, from the viewpoint of Newton's Classical Mechanics, the total gravitational energy $\mathscr{E}_{\mathrm{T}}$ of an isolated system is basically defined as the straight summation of the translational kinetic energy possessed by a test object with the constituent potential energy of the system; e.g.:

$$\mathscr{E}_{\mathrm{T}} = \left(\frac{1}{2} m v^2\right) - \left(\frac{GMm}{r}\right) , \tag{1}$$

where, in the given two-body scenario, $M$ is the immobile host mass, and $m$ the rest mass of the test object possessing the instantaneous velocity $v$ in the former's proximity, with $G$ being the known gravitational constant (that we had shown in [1] to inevitably and conformally vary vis-à-vis the change in the strength of gravity). Note further that the "total energy" $\mathscr{E}_{\mathrm{T}}$ written above is a negative quantity; meaning that one has to deliver energy to $m$ hosted by $M$ in order to dislodge it from its orbit to infinity away.

Assigning — *according to familiar nomenclature and per commutativity* — the first term on the RHS between parantheses the letter $\mathcal{K}$ (i.e., "kinetic energy"), and the second term that follows after (including the minus sign) the letter $\mathcal{U}$ (i.e., "potential energy"), we can rewrite Eq. (1) in the form of:

$$\mathscr{E}_{\mathrm{T}} = \mathcal{K} + \mathcal{U} ; \tag{2}$$

yet, without any relativistic considerations (not to mention any alteration vis-à-vis the test mass brought about by gravitation) up to this point.

To arrive at a comparable conserved quantity under the formalism of general theory of relativity (GTR; cf. [20, p. 250-252]), one would instead have to start with

$$\mathscr{E}_{\mathrm{GTR}} = \frac{mc^2\sqrt{g_{00}}}{\sqrt{1-\dfrac{v^2}{c^2}}}\,, \tag{3}$$

where the velocity $v$ is to be measured by an observer resting on the test object's trajectory as it passes by, and where the squarerooted metric tensor element $\sqrt{g_{00}}$ is manifestly identical to $\sqrt{1+(2\phi/c^2)} = \sqrt{1-2\alpha}$ (with $\phi$ being the Newtonian gravitational potential $-\dfrac{GM}{r}$ in the "limiting case" and $\alpha$ similarly being $\dfrac{GM}{rc^2}$); insofar as carrying on with the derivations to finally land at what essentially resembles Eq. (2):

$$\mathscr{E}_{\mathrm{T}} \approxeq \mathscr{E}_{\mathrm{GTR}} - (mc^2)\,. \tag{4}$$

Eventually — since $\sqrt{1-\dfrac{v^2}{c^2}} \approxeq 1-\dfrac{1}{2}\dfrac{v^2}{c^2}$ ··· up to a second order Taylor expansion, and because $\sqrt{1-\xi} \approxeq 1-\dfrac{\xi}{2}$ alongside $\dfrac{1}{1-\xi} \approxeq 1+\xi$ (for $\xi \ll 1$) — we shall, in the weak gravitational limit and with respect to non-relativistic velocities, get:

$$\sqrt{g_{00}} \approxeq \sqrt{1-2\alpha} \approxeq 1-\frac{\not{2}\alpha}{\not{2}} = (1-\alpha)\,, \tag{5a}$$

$$\mathscr{E}_{\mathrm{GTR}} \approxeq \frac{mc^2\sqrt{1-2\alpha}}{\sqrt{1-\dfrac{v^2}{c^2}}}\,, \tag{5b}$$

$$\mathscr{E}_{\mathrm{GTR}} \approxeq \frac{mc^2\;(1-\alpha)}{1-\dfrac{1}{2}\dfrac{v^2}{c^2}}\,, \tag{5c}$$

$$\mathscr{E}_{\mathrm{GTR}} \approxeq mc^2\,(1-\alpha)(1+\frac{1}{2}\frac{v^2}{c^2})\,, \tag{5d}$$

$$\mathscr{E}_{\mathrm{GTR}} \approxeq mc^2\,(1-\alpha+\frac{1}{2}\frac{v^2}{c^2})\,; \tag{5e}$$

where one may neglect $(-\alpha \, \dfrac{v^2}{2c^2})$ for having an inconsequential magnitude, and which well agrees with the Newtonian approximation due to the fact that

$$\mathscr{E}_{\mathrm{GTR}} \approxeq mc^2 - \left(\frac{GMm\cancel{c^2}}{r\cancel{c^2}}\right) + \left(\frac{1}{2}\,\frac{m\cancel{c^2}v^2}{\cancel{c^2}}\right), \tag{6a}$$

$$\mathscr{E}_{\mathrm{GTR}} - mc^2 \approxeq -\left(\frac{GMm}{r}\right) + \left(\frac{1}{2}\,mv^2\right), \tag{6b}$$

$$\mathscr{E}_{\mathrm{T_{GTR}}} \approxeq \mathcal{U} + \mathcal{K}\,. \tag{6c}$$

It shall soon be seen that such an outcome, from the general relativistic formalism of Eq. (3) leading all the way to Eq. (6c) under the aforesaid circumstances, will be the same as what *Yarman's Approach* furnishes up to a third order Taylor expansion; except that the additional terms including the third term will start to diverge from each other. Strikingly enough, the miniscule difference in question serves to substantially change our picture of the universe, where singularities of any kind (be they "purely mathematical" or "in possession of a physical counterpart") altogether vanish.

From the already provided references at the beginning of the text, it can be seen that our "overall relativistic energy" — or, just the same, "total relativistic energy" — for especially a stationary field (as was the case too for general relativity above) can be expressed as

$$\mathscr{E}_{\mathrm{YARK}} = \frac{mc^2\,\exp\left(-\alpha\right)}{\sqrt{1-\dfrac{v^2}{c^2}}}\,, \quad \text{leading to} \tag{7a}$$

$$\mathscr{E}_{\mathrm{T}} \approxeq \mathscr{E}_{\mathrm{YARK}} - (mc^2)\,; \tag{7b}$$

with the velocity $v$ in the denominator of Eq. (7a) to be universally measured from any plausible location regardless, while the utmost theoretical barrier $c$ for the speed of light in empty space truly remains a ubiquitous constant under correspondent "composite scale transformations" for any ponderable field in full harmony with Lorentz Invariant combinations. Also, needless to repeat, $\alpha$ represents the customary $\dfrac{GM}{rc^2}$, where, in the given two-body scenario, $M$ happens to be the immobile host mass, while $r$ is the distance between said host body and the test particle as scrutinized by the distant observer.

Note nevertheless that the fourth co-author had arrived at his Eq. (7a) in only a few lines by taking into account solely the law of energy conservation embodying the mass and energy equivalence of the special theory of relativity (STR); whereas, the harvesting of Eq. (3) out of the framework of GTR requires, at the very least, much more cumbersome mathematical tools such as tedious calculations regarding the curvature of space-time and related metric operations.

Since, once again, $\sqrt{1-\frac{v^2}{c^2}} \approxeq 1-\frac{1}{2}\frac{v^2}{c^2} \cdots$ up to a second order Taylor expansion, and because $\frac{1}{1-\xi} \approxeq 1+\xi$ alongside $\exp(-\xi) \approxeq 1-\xi$ (for $\xi \ll 1$), the aforesaid YARK overall energy ($\mathscr{E}_{\text{YARK}}$) can be straightforwardly outlined as follows:

$$\mathscr{E}_{\text{YARK}} \approxeq \frac{mc^2\,(1-\alpha)}{1-\frac{1}{2}\frac{v^2}{c^2}}\,, \tag{8a}$$

$$\mathscr{E}_{\text{YARK}} \approxeq mc^2\,(1-\alpha)(1+\frac{1}{2}\frac{v^2}{c^2})\,, \tag{8b}$$

$$\mathscr{E}_{\text{YARK}} \approxeq mc^2\,(1-\alpha+\frac{1}{2}\frac{v^2}{c^2})\,; \tag{8c}$$

yielding

$$\mathscr{E}_{\text{YARK}} \approxeq mc^2 - \left(\frac{GMm\cancel{c^2}}{r\cancel{c^2}}\right) + \left(\frac{1}{2}\frac{m\cancel{c^2}v^2}{\cancel{c^2}}\right), \tag{9a}$$

$$\mathscr{E}_{\text{YARK}} - mc^2 \approxeq -\left(\frac{GMm}{r}\right) + \left(\frac{1}{2}mv^2\right), \tag{9b}$$

$$\mathscr{E}_{\text{T}_{\text{YARK}}} \approxeq \mathcal{U} + \mathcal{K}\,, \tag{9c}$$

owing to the fact that the algebraic operation with respect to $(-\alpha \frac{v^2}{2c^2})$ during the passage from Eq. (8b) to Eq. (8c) remains yet again non-significative for weak gravity regimes (viz., it is in the order of $\frac{v^4}{c^4}$; which can be confidently ignored altogether).

Seeing as Eq. (4) and Eq. (7b) are the same and remarkably well agree with Newtonian mechanics in the limit of a weak gravitational field as shown by Eq. (6c) and Eq. (9c), all that remains is to spell out how a third order Taylor expansion exercise will lead to a divergence in between the predictions of YARK theory and GTR. The fundamental departure point of these two approaches lies in the difference between the presence of a squareroot term as elucidated in Eq. (5b) in the case of GTR and the presence of an exponential term in YARK theory as shown in Eq. (7a). In other words, "singularities" espoused by general relativity as a consequence of the aforementioned squareroot term are obliterated due to its direct replacement with an exponential term in YARK theory as implied by the law of energy conservation embodying the mass and energy equivalence of STR.

One additional detail to consider is that a velocity in gravitation, under the framework of GTR, turns out to be lesser when assessed by the distant observer as compared to its value as attested to by the local observer. Light, in particular, slows down according to GTR as such, although its velocity had been propounded to be constant in STR beforehand. In contradistinction, YARK theory posits that the utmost theoretical speed barrier $c$ for any material object, including photons, remains constant no matter what (although it may take an infinite amount of energy to asymptotically approach that barrier). In fact, all given velocities — *be they in gravitation or some other field* — remain untouched in YARK regardless of who measures them (as shall be elaborated on in the Conclusion Section).

Proceeding from this stage onward, it is easy to disclose the third term of the Taylor series pertaining to both approaches that stands to change everything with respect to the established understanding of the cosmos.

## 2. A new cosmology without "Singularities": Our third order Taylor expansion exercise as regards the "Total Relativistic Energy" equations of respectively GTR and YARK theory

According to our exposition so far, we have elucidated how YARK theory and GTR both coincide with Newton's Classical Mechanics in the "limiting case", and further hinted at how a continued Taylor expansion exercise will yield divergent results. Let us now focus on what GTR furnishes with respect to the third terms in the Taylor series starting from Eq. (3) when it is rephrased in order to highlight the fact that the test mass $m_{0\infty}$ (which we had called $m$ up to this point for simplicity's sake) is to be assessed by an infinitely distant observer in his own frame of reference:

$$\mathscr{E}_{\text{GTR}} = \frac{m_{0\infty} c^2 \sqrt{1-2\alpha}}{\sqrt{1-\dfrac{v^2}{c^2}}} \,, \tag{10a}$$

$$\mathscr{E}_{\text{GTR}} \approxeq \frac{m_{0\infty} c^2 \left(1 - \dfrac{\cancel{2}\alpha}{\cancel{2}} - \dfrac{1}{2}\dfrac{\cancel{4}\alpha^2}{\cancel{8}}\right)}{1 - \dfrac{1}{2}\dfrac{v^2}{c^2} - \dfrac{1}{8}\dfrac{v^4}{c^4}} \,, \tag{10b}$$

$$\mathscr{E}_{\text{GTR}} \approxeq m_{0\infty} c^2 \left(1 - \alpha - \frac{1}{2}\alpha^2\right) \left(\frac{1}{1 - \dfrac{1}{2}\dfrac{v^2}{c^2} - \dfrac{1}{8}\dfrac{v^4}{c^4}}\right) \,, \tag{10c}$$

$$\mathscr{E}_{\text{GTR}} \approxeq m_{0\infty} c^2 \left(1 - \alpha - \frac{\alpha^2}{2}\right) \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4}\right) \,. \tag{10d}$$

At this juncture, one may pursue the mathematical task at hand with respect to, say, Earth in a (nearly) circular orbit around the Sun (in which case, both $\alpha$ and $\frac{v^2}{c^2}$ are of the order of $10^{-8}$). We can therefore display the orders of magnitudes of different quantities coming into play:

$$\begin{aligned} \mathscr{E}_{\text{GTR}} \approxeq m_{0\infty} c^2 \Bigg(1 - & \overset{[10^{-8}]}{\alpha} - \overset{[10^{-16}]}{\frac{\alpha^2}{2}} \Bigg| \\ & \Bigg| + \frac{1}{2}\frac{v^2}{c^2} - \frac{\alpha}{2}\frac{v^2}{c^2} \Bigg| \overset{[10^{-24}]}{- \frac{1}{4}\frac{\alpha^2 v^2}{c^2}} \\ & + \frac{3}{8}\frac{v^4}{c^4} \Bigg| - \overset{[10^{-24}]}{\frac{3\alpha}{8}\frac{v^4}{c^4}} - \overset{[10^{-32}]}{\frac{3\alpha^2}{16}\frac{v^4}{c^4}} \Bigg) \,, \end{aligned} \tag{11a}$$

$$\mathscr{E}_{\text{GTR}} \approxeq m_{0\infty} c^2 \left(1 - \alpha + \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha\,\frac{v^2}{c^2}\right) \,; \tag{11b}$$

where we have omitted the terms in the third row of Eq. (11a) denoting the Earthbound magnitudes of especially $10^{-24}$ and $10^{-32}$ for their diminutive substance under even mildly strong gravity.

We can henceforth expand the $\alpha = \dfrac{GM}{rc^2}$ terms of Eq. (11b) to find the additional kinetic and potential energy components that should accompany Eq. (6c) when aiming for higher precision as compared to a simple Newtonian approximation:

$$\mathscr{E}_{\text{GTR}} \approxeq m_{0\infty}c^2 \left[ \left( 1 - \alpha + \frac{1}{2}\frac{v^2}{c^2} \right) - \frac{1}{2}\alpha \left( \alpha + \frac{v^2}{c^2} \right) \right] , \tag{12a}$$

$$\begin{aligned} \mathscr{E}_{\text{GTR}} \approxeq & \left[ mc^2 - \left( \frac{GMm\cancel{c^2}}{r\cancel{c^2}} \right) + \left( \frac{1}{2} \frac{m\cancel{c^2}v^2}{\cancel{c^2}} \right) \right] \\ & \boxed{ - \cancel{\left[ \left( \frac{G^2M^2m\cancel{c^2}}{2r^2\cancel{c^4}} \right) - \left( \frac{GMm\cancel{c^2}v^2}{2r\cancel{c^4}} \right) \right]} } \\ & - \frac{1}{2} \frac{GMm\cancel{c^2}}{r\cancel{c^2}} \left[ \left( \frac{GM\,m\cancel{c^2}}{r\cancel{c^2}\boxed{mc^2}} \right) + \left( \frac{m\cancel{c^2}v^2}{\cancel{c^2}\boxed{mc^2}} \right) \right] ; \end{aligned} \tag{12b}$$

which, via Eq. (4), yields

$$\mathscr{E}_{\text{T}_{\text{GTR}}} \approxeq \mathcal{U} + \mathcal{K} + \frac{\mathcal{U}}{2} \left( \frac{-\mathcal{U}}{mc^2} + \frac{2\mathcal{K}}{mc^2} \right) , \tag{13a}$$

$$\mathscr{E}_{\text{T}_{\text{GTR}}} \approxeq \mathcal{U} + \mathcal{K} - \frac{\mathcal{U}^2}{2mc^2} + \frac{\cancel{2}\mathcal{U}\mathcal{K}}{\cancel{2}mc^2} , \tag{13b}$$

$$\mathscr{E}_{\text{T}_{\text{GTR}}} \approxeq \mathcal{U} + \mathcal{K} - \frac{\mathcal{U}^2}{2mc^2} + \frac{\mathcal{U}\mathcal{K}}{mc^2} \tag{13c}$$

for greater gravitational strengths in comparison to the run-of-the-mill situation with our Solar System. Note once again, the test particle's GTR denomination $m_{0\infty}c^2$ we used throughout entails that this is to be assessed by an infinitely remote observer in his reference frame, which is a quantity found if such a rest energy was weighed at a location far removed from all sources of gravitation. As it had been deliberated many times over in the relevant publications by the fourth co-author and his colleagues, *the term for the "rest mass"* (or the same, *"rest energy"*, if $c$ were unity) *of the object weighed at infinity* under both GTR and the framework of YARK drops off of their respective equations of motion at the end come what may; which thence ensures — particularly in the case YARK — full compatibility with the *Weak Equivalence Principle* (WEP).

At this time, let us concentrate on how YARK theory differs from GTR when the third terms in the Taylor series are harnessed with regards to our enterprise:

$$\mathscr{E}_{\mathrm{YARK}} = \frac{m_{0\infty}c^2 \exp\left(-\alpha\right)}{\sqrt{1-\dfrac{v^2}{c^2}}} \,, \tag{14a}$$

$$\mathscr{E}_{\mathrm{YARK}} \approxeq \frac{m_{0\infty}c^2 \left(1-\alpha+\dfrac{1}{2}\alpha^2\right)}{1-\dfrac{1}{2}\dfrac{v^2}{c^2}-\dfrac{1}{8}\dfrac{v^4}{c^4}} \,, \tag{14b}$$

$$\mathscr{E}_{\mathrm{YARK}} \approxeq m_{0\infty}c^2 \left(1-\alpha+\frac{1}{2}\alpha^2\right)\left(\frac{1}{1-\dfrac{1}{2}\dfrac{v^2}{c^2}-\dfrac{1}{8}\dfrac{v^4}{c^4}}\right) \,, \tag{14c}$$

$$\mathscr{E}_{\mathrm{YARK}} \approxeq m_{0\infty}c^2 \left(1-\alpha+\frac{\alpha^2}{2}\right)\left(1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}\right) \,. \tag{14d}$$

Were we, once again, to pursue the undertaking at hand with respect to, say, Earth in a circular orbit around our Sun, it would become possible to readily display the orders of magnitudes of different quantities coming into play:

$$\begin{aligned}\mathscr{E}_{\mathrm{YARK}} \approxeq m_{0\infty}c^2 \Bigg(1 - \overset{[10^{-8}]}{\alpha} + \overset{[10^{-16}]}{\frac{\alpha^2}{2}} \Bigg| \\ \Bigg| + \frac{1}{2}\frac{v^2}{c^2} - \frac{\alpha}{2}\frac{v^2}{c^2} \Bigg| \overset{[10^{-24}]}{+\frac{1}{4}\frac{\alpha^2 v^2}{c^2}} \\ + \frac{3}{8}\frac{v^4}{c^4} \Bigg| - \overset{[10^{-24}]}{\frac{3\alpha}{8}\frac{v^4}{c^4}} + \overset{[10^{-32}]}{\frac{3\alpha^2}{16}\frac{v^4}{c^4}} \Bigg) \,,\end{aligned} \tag{15a}$$

$$\mathscr{E}_{\mathrm{YARK}} \approxeq m_{0\infty}c^2 \left(1-\alpha+\frac{1}{2}\frac{v^2}{c^2}+\frac{1}{2}\alpha^2-\frac{1}{2}\alpha\frac{v^2}{c^2}\right) \,; \tag{15b}$$

where we have once more omitted the terms, this time, in the third row of Eq. (15a) denoting the Earthbound magnitudes of especially $10^{-24}$ and $10^{-32}$ for their diminutive substance under even mildly strong gravity.

One can thereby expand yet again the $\alpha = \dfrac{GM}{rc^2}$ terms of Eq. (15b) to find the additional kinetic and potential energy components that should accompany, this time, Eq. (9c) when aiming for higher precision as compared to a simple Newtonian approximation:

$$\mathscr{E}_{\mathrm{YARK}} \approxeq m_{0\infty}c^2 \left[ \left( 1 - \alpha + \frac{1}{2}\frac{v^2}{c^2} \right) + \frac{1}{2}\alpha \left( \alpha - \frac{v^2}{c^2} \right) \right] , \tag{16a}$$

$$\begin{aligned} \mathscr{E}_{\mathrm{YARK}} \approxeq & \left[ mc^2 - \left( \frac{GMm\cancel{c^2}}{r\cancel{c^2}} \right) + \left( \frac{1}{2}\frac{m\cancel{c^2}v^2}{\cancel{c^2}} \right) \right] \\ & \boxed{+ \left[ \cancel{\left( \frac{G^2M^2m\cancel{c^2}}{2r^2c^{\cancel{4}\,2}} \right) - \left( \frac{GMm\cancel{c^2}v^2}{2rc^{\cancel{4}\,2}} \right)} \right]} \\ & + \frac{1}{2}\frac{GMm\cancel{c^2}}{r\cancel{c^2}} \left[ \left( \frac{GM\,m\cancel{c^2}}{r\cancel{c^2}\boxed{mc^2}} \right) - \left( \frac{m\cancel{c^2}v^2}{\cancel{c^2}\boxed{mc^2}} \right) \right] ; \end{aligned} \tag{16b}$$

which, via Eq. (7b), then yields

$$\mathscr{E}_{\mathrm{T_{YARK}}} \approxeq \mathcal{U} + \mathcal{K} - \frac{\mathcal{U}}{2} \left( \frac{-\mathcal{U}}{mc^2} - \frac{2\mathcal{K}}{mc^2} \right) , \tag{17a}$$

$$\mathscr{E}_{\mathrm{T_{YARK}}} \approxeq \mathcal{U} + \mathcal{K} + \frac{\mathcal{U}^2}{2mc^2} + \frac{\cancel{2}\mathcal{U}\mathcal{K}}{\cancel{2}mc^2} , \tag{17b}$$

$$\mathscr{E}_{\mathrm{T_{YARK}}} \approxeq \mathcal{U} + \mathcal{K} + \frac{\mathcal{U}^2}{2mc^2} + \frac{\mathcal{U}\mathcal{K}}{mc^2} \tag{17c}$$

for stronger gravitational pools in comparison to the daily situation with our Sun, Earth, and neighboring planets.

Contrasting YARK theory's total gravitational energy Eq. (17c) with GTR's Eq. (13c) reveals the critical difference to occur in the sign of their respective third terms: It is negative in the case of GTR, but positive in the case of YARK.

## 3. Conclusion

When our preceeding third order Taylor expansion exercise culminated with the "additive serialism" in YARK theory's total gravitational energy Eq. (17c) — as compared to the "alternant serialism" in the correspondent Eq. (13c) obtained under the formalism of GTR — the successive summation of solely positive terms thence seen betokens a whole new cosmology. As a matter of fact, if one remains faithful to the "limiting case" (e.g., Newtonian Classical Mechanics), regular outcomes with respect to centuries-long secular experiments and observations are already acquired with *Yarman's Approach*. On the other hand, should one continue with the third terms in the relevant Taylor series, not only are later centennial features such as the precession of the perihelion of Mercury, Shapiro delay, gravitational redshift, gravitational lensing, Pound & Rebka results, etc... (all historically attributed to the "success" of GTR) are harvested via YARK theory [*cf.* 9, 10, 11, 12, 15], but one eventually comes to behold a universe without any *warping* of the "fabric of space-time" by masses or any *ripples* therein such as "gravitational waves" (GWs). More notably, the new cosmic outlook presented by YARK starting from Eq. (14a) all through Eq. (17c) makes certain that any sort of singularity — be it "purely mathematical" or "in possession of a physical counterpart" — vanishes forthwith.

Let us henceforward revisit the critical YARK theory and GTR equations pertaining to the "total gravitational energy of a two-body system" by also taking into account the previously neglected $+\dfrac{3v^4}{8c^4}$ terms (of the demarcated $10^{-16}$ order of magnitude in the case of our home planet circularly revolving around its host star) from the third rows of respectively YARK's Eq. (15a) and GTR's Eq. (11a):

$$\mathscr{E}_{\mathrm{T_{YARK}}} \approxeq \mathcal{U}+\mathcal{K}+\frac{\mathcal{U}^2}{2mc^2}+\frac{\mathcal{U}\mathcal{K}}{mc^2}+\mathcal{K}\frac{3v^2}{4c^2} = \mathcal{K}\left(1+\frac{3\mathcal{K}}{2mc^2}\right)+\mathcal{U}\left(1+\frac{\mathcal{U}+2\mathcal{K}}{2mc^2}\right) \quad (18a)$$

$$\mathscr{E}_{\mathrm{T_{GTR}}} \approxeq \mathcal{U}+\mathcal{K}-\frac{\mathcal{U}^2}{2mc^2}+\frac{\mathcal{U}\mathcal{K}}{mc^2}+\mathcal{K}\frac{3v^2}{4c^2} = \mathcal{K}\left(1+\frac{3\mathcal{K}}{2mc^2}\right)+\mathcal{U}\left(1-\frac{\mathcal{U}-2\mathcal{K}}{2mc^2}\right) \quad (18b)$$

Although further deliberations can be made on the distinguishing characteristics of these two equations with respect to mundane or extra-mundane physics, the abovegiven exposition should suffice to clarify what is meant by "*overall relativistic energy*" under particularly the framework of *Yarman's Approach* or its generalized extension called *YARK theory of gravity* from the initials of its principal founders, and how both YARK's Eq. (18a) and GTR's Eq. (18b) diverge from the Newtonian Eq. (1) and Eq. (2).

In retrospect, the following nuance ought to be stressed to shed more light on the situation pertaining to how all velocities in YARK theory, and especially the utmost theoretical speed barrier $c$ for any material object, shall remain Lorentz Invariant: The reason for this is rooted in Quantum Mechanics (QM) with which *Yarman's Approach* (and, by extension, YARK theory) is in full harmony. Specifically, when the rest mass of an object — say, an Hydrogen atom embedded in gravitation vis-à-vis the gravitational binding energy coming into play — is made to decrease owing to the law of energy conservation embodying the mass and energy equivalence of the STR insofar as the mass decrease of concern is injected into the quantum mechanical description of the entity in question, the object's size will get inflated uniformly as much, while its internal energy will become sluggish as much (e.g., its temporal rate shall lessen conformally). In other words, its size and periodicity are to be stretched by the same amount in YARK when said object is set in a gravitational environment, which turns out to be commensurate with the binding energy the entity at hand cedes under gravity. Such is, in point of fact, why velocities — and particularly the theoretical upperbound $c$ for the speed of light in vacuum — are indeed unaltered within the framework YARK.

All of the above ensures an outstanding simplicity in comparison to the formalism of GTR. Note further that the addition of velocities in gravitation still follows the established rules of special relativity when working with either YARK theory or *Yarman's Approach* that constitutes our novel gravitational framework's roots, which is extensible to all force fields while maintaining total compliance with Quantum Mechanics.

We leave intricate mathematical analyses of our derivations and results for future studies as well as other interested researchers, yet feel satisfied that a rather obscure feature pertaining to "*overall relativistic energy*" alongside "*total gravitational energy*", in contrast to Newtonian Classical Mechanics, has been explained in a directly comparative and clearly understandable way — despite the fact that the canonical formulation of Einstein's GTR does not seem to allow an interpretation which makes field energy localizable unto bodies. Nevertheless, we believe we have demonstrated herein that such an interpretation shall still yield valid observational results, insofar as remaining in force to permit a straight juxtaposition with *Yarman's Approach* when it has been extended to the level of YARK theory of gravity.

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## . Comments by Reviewers and Our Answers to them .

REVIEWER 1

The article provides reasonable arguments in favor of an alternative theory of gravity.

It is well known that the modern theory of gravity — GTR has drawbacks. This is primarily the presence of a singularity in the solutions and the absence of local laws of conservation of energy-momentum.

The authors drew attention to the conservation laws and their conclusions are based on plausible reasoning. In general, the article makes a good impression.

An important result is the transition from formulas (5b) and (5c) to formula (7a). It should be noted that a similar transition was made by Einstein in 1907 (Einstein A. *Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen*. Jahrb. D. Radioaktivitat u. Elektronik, 4, 411-462 (1907)). It would be reasonable to refer to this result.

Need to carefully check the text. So in the formula (7a) a typo was made. Instead of $\exp^{(-\alpha)}$, you need to write either $\exp(-\alpha)$ or $e^{(-\alpha)}$.

In general, the article should be of interest to the reader. We can conclude that it is quite worthy of publication.

REPLY TO REVIEWER 1

We thank the reviewer cordially for his impartial scrutinization both towards GTR and YARK theory of gravity.

As is well known, GTR's constraint is to meet the Newtonian approach at the weak gravitational regime, and hence, with respect to low velocities for the client object. Therefore, we believe we need no further reference on this other than the source that Eq. (5b) was borrowed from (i.e., [20]).

We re-checked the text carefully and indeed corrected the typographical error in Eq. (7a) as well as in similar places.

REVIEWER 2

I have read the paper of Mr. Yarman and colleagues and I found no mathemetical error. However, the physical point of view seems insufficient: What is the referential used? How it is defined? How the proper time, the time of aboard atomic clocks, is related to the motion of the space vehicle and to the gravitational field? What is the meaning of the following phrase ...: "*For instance, a velocity in gravitation turns out to be lesser when assessed by the distant observer as compared to its value as assessed to by the local observer*". A velocity is not an intrisic property, it a space divided by a time in a given, well defined referential...

I agree that YARK theory removes the singularity that appear in the Schwarzschild gravitational field at the distance $r = 2m$ and that Schwarzschild singularity is only an artificial singularity, the very simple Painlevé transformation of the cosmical time was the first exemple of this removal.

REPLY TO REVIEWER 2

We thank the referee for this meticulous review.

We brought rigor to our sentence he righteously indicated in the article. Said sentence now reads as:

— *One additional detail to consider is that a velocity in gravitation, under the framework of GTR, turns out to be lesser when assessed by the distant observer as compared to its value as attested to by the local observer.*

Thereby, it would be useful to recall that we have two fundamental frames of reference in the present approach:

i) The frame of the distant observer, and
ii) That of the observer at rest in gravitation.

From this standpoint, below are the explicit definitions of various crucial concepts brought to attention by our referee...

**What is the referential used?**

— It is, in particular, the reference frame of the distant observer if no further specification is made in the text.

**How it is defined?**

— The necessary specification is provided in the text. The distant observer is practically infinitely far away from the gravitational pool.

**How the proper time, the time of aboard atomic clocks, is related to the motion of the space vehicle and to the gravitational field?**

— This is definitely an appropriate question. All the same, the article aimed to analyze comparatively how, in the observational frame of the distant observer, "overall energy" will be viewed both in GTR and YARK theory of gravity. Details with respect to the particular question of the reviewer is provided in Yarman's and also in Yarman et al.'s articles cited in the manuscript. Nevertheless, we can provide a direct answer to the question of the reviewer, for, "the proper time in gravitation will vary as viewed by the distant observer in exactly the same way as that delineated by the overall energy". Now, how things will be assessed by a local observer at rest is, we believe, something to be considered apart and remains beyond the scope of this paper...

We totally agree with the referee as to the fact that the "**Schwarzschild singularity is only an artificial singularity, and the very simple Painlevé transformation of the cosmical time was the first exemple of its removal**". All the same, we are of the opinion that this peculiarity goes beyond the limits adopted by the present contribution, because we exclusively dwelt on the "singularity conceptualization" held by the mainstream.

OUR CORRESPONDENCE WITH REVIEWER 3

For historical and aesthetical reasons, we do not seperate our third referee's comments from our answers to him.

My first impression is that equation (3) is self-contradictory.

This, as shall be delineated below, is a pure GTR Equation that we borrowed from Landau & Lifshitz [cf., Eq. (88.9)]. Reference to their book is provided in the manuscript. So, it is hard to believe that it is in any way inappropriate vis-à-vis GTR.

In it supposedly $v$ is constant and yet you take the coefficient of the time component of the Schwarzschild metric which contains the gravitational field on the assumption of an asymptotically weak field. Also recall in general relativity, gravity is not a force which does work so it is relegated to the time component of the metric instead of the space component. In special relativity the velocity is constant and you get the time dilation term. The two don't mix.

Eqn (7a) is also perplexing because you are considering a cut-off for the gravitational force a la Laplace. Why? What is the motivation for this?

This is YARK's fundamental equation. It is written in a few lines only. Our reviewer can check it here: `http://aflb.ensmp.fr/AFLB-293/aflb293m137.htm`. Reference to "*The general equation of motion via the special theory of relativiy and quantum mechanics*" accessible from said link, as well as all the related material, is provided in [10].

The problem is what does Eq (88.9) of Landau & Lifshitz mean.

As the text shows, it is the invariant of the motion.

It comes right from the Schwartzschild metric...

This says, on the whole:

i) The internal energy $mc^2$ of an object brought to a gravitational field is redshifted due to "curvature"; and this, as much as $\sqrt{(1-2\alpha)}$, where $\alpha$ is, as usual, $\dfrac{GM}{rc^2}$.

ii) If the object is further brought to a motion whose instantaneous velocity is $v$, then the previously decreased energy is dilated as much as the Lorentz coefficient that comes into consideration and, the way the calculations lead to, as referred to by the local observer at rest...

iii) Therefore, the overall energy $\mathscr{E}$ of the original object of mass $mc^2$ becomes, on the whole, $\mathscr{E} = \dfrac{mc^2\sqrt{(1-2\alpha)}}{\sqrt{1-\dfrac{v^2}{c^2}}}$ (i.e., Eq. (5b) in the text).

iv) And this (for a closed system) stays invariant throughout the motion...

If it is uniform translation then it is clearly wrong. The gravitational potential tends to accelerates particles so $v$ cannot be constant. Rather, if it is uniform rotation

No!..

that is a different matter entirely. Historically, this goes back to the uniformly rigidly rotating disk treated by Ehrenfest and Einstein. Is there contraction in the direction of rotation?

In GTR, yes... But not in YARK...

If so you need more rulers to measure the circumference because the rulers undergo contraction in the direction of rotation.

This proved to be the Achilles' heel of Einstein's theory. Uniform acceleration is not equivalent to a gravitational potential.

Einstein originally stated that the effect of rotational acceleration is the same as the effect of gravitation corresponding to the same accelerational intensity as that of rotation...

In fact, the uniformly rotating disk corresponds to the hyperbolic plane where the relevant metric is the Beltrami metric. The Beltrami metric is conformally equivalent to the spatial part of the Schwazschild metric, as I show in my book "*A New Perspective on Relativity*".

We would like to suggest that our reviewer foregoes the rotating disc (which the fourth co-author and his colleagues worked on enormously), and that it is better to concentrate on the solution of Einstein's Field Equations and their Schwartzschild solution that yields the Landau & Lifshitz equation he referred to.

Since the uniformly rotating disc corresponds to the hyperbolic plane, it is well-known that an observed placed any where on the disc (a Poincarite) cannot distinguish his position by using his measuring sticks or clocks since they expand or shrink with him. Therefore, Einstein was wrong when he said that a clock placed on the rim of the disk would go slower than one placed at the center of the disc which would correspond to an inertial system. The whole problem of contraction is ill-posed and highlights the inadequacy of Einstein's equivalence principle. It is now argued that the equivalence principle holds "locally". A principle holds or doesn't hold so the adjective is superfluous.

OK...

Equation (88.9) was derived from the Hamilton-Jacobi equation. That equation contains two masses, a mass $m$ moving with velocity $v$ and a mass that creates the gravitational potential $\phi$. It is known that general relativity has failed to solve the two body problem. Thus, the Hamilton-Jacobi equation cannot be equivalent to Einstein's equations.

OK...

Now here is the rub. Gravity, in Einstein's theory, is not a force that does work by displacing the particle through a given distance. Therefore, it does not belong in the energy-stress tensor. Moreover, energy cannot be localized in general relativity. I therefore ask how can (88.9) be a conserved quantity?

We invite our referee to once more look in Landau-Lifshitz (L-L). While this solution of GTR is not well-known, it is still there and is well applicable.

Since gravity does no work it cannot be part of the space component of the metric. It is therefore relegated to the time component of the metric as (88.9) clearly shows. Then it should show up in the energy which is does. But, doesn't that contradict the fact that the energy-momentum tensor be devoid of all aspects of gravity?

If gravity is accounted for by Einstein's tensor $G$, how does his $T$ tensor know that? What does $G = -kT$ mean if the two tensors apply to different situations. This was questioned by Silberstein in 1936 where he considered $T$ as describing a perfect fluid while $G$ refers to a media with an index of refraction. This goes back to Eddington's old idea that the affect that gravity has on light is analogous to the passage of light through a medium with an index of refraction different than the vacuum. (I personally agree with such an interpretation and the effect that a static gravitational field would have on light.) The tensor $T$ would be independent of the index of refraction and hence Einstein's equation would be tantamount to equating apples to oranges.

If the Hamilton-Jacobi equation would be equivalent to Einstein's equations then it would save a lot of time and effort. It would also show that only the ray nature of light or particle motion would apply. This would have devastating consequences on the wave nature of the linearized Einstein equations representing gravitational waves. Moreover, it is claimed that the pseudo-tensor is the source of gravitational waves. But, a pseudo-tensor can be made to disappear by a mere change in the coordinates. And if general relativity hasn't solved the two body problem how can numerical relativity describe the merger of two black holes?

We once again invite our referee to consult Landau-Lifshitz (L-L).

Regarding black holes it is said that for distances within the Schwarzschild radius, the (outer) solution of the Schwarzschild metric is turned "inside out" where time and space swap roles. Why the would Schwarzschild gone through all the trouble of deriving his "inner" solution? And why is there no mention of black holes in the outer solution? The answer is simple: You can't extend the metric beyond the Schwarzschild radius, that corresponds to the radius of the rim of the disc. Since the metric does not have constant curvature, the disc itself is described by the inner metric which does have constant curvature. The transition between the two is affected by Kepler's III. law. And that law was used by Hulse and Taylor to calculate the gravitational wave luminosity derived a decade and a half earlier by Peters and Mathews. Their article was entitled "*Gravitational radiation from point masses in a Keplerian orbit*". But, if radiation does occur does that destroy the idea of a Keplerian orbit? This would be analogous to the quantum mechanical problem of an orbiting electron radiating and falling into the nucleus.

All these are mechanical manipulations, of which Landau & Lifshitz were masters at. One has to (or should) ask what do these manipulations mean physically?

It means, physically, what the fourth co-author wrote at the very beginning of his theoretical enterprise, and it really shows a remarkable parallel with our own diabolically similar equation $\mathscr{E} = \dfrac{mc^2 \exp(-\alpha)}{\sqrt{1-\dfrac{v^2}{c^2}}}$ (i.e., Eq. (7a) in the text) which he derived some 20 years ago in only a few lines and while not even knowing anything about the L-L equation by then.

Thus, we encourage our referee to please scrutinize the reference bearing the title "*The general equation of motion via the special theory of relativiy and quantum mechanics*" that had been given in [10].

What we tackle in our current contribution just so happens to be the parallelism of concern...

In any case, we are indebted to our reviewer so much for the time and effort he has bestowed on us and our work.